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CONTENTS

1. D.C. AMMETERS AND VOLTMETERS

Reference: APPLIED ELECTRICITY 1, "D.C. Measurements", Sections 1-3.

1. METER SCALES.

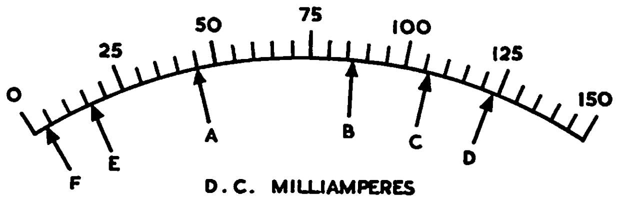
1.1 An ammeter is used in an electrical circuit to measure the current in units of amperes. For lower values, milliammeters and microammeters are used.

A voltmeter measures the voltage in units of volts. Kilovoltmeters and millivoltmeters are also used.

1.2 The entire range of a meter scale is divided into a number of divisions. The main divisions are usually numbered, but some numbers may be omitted to make the dial easier to read. Each main division is divided into a number of smaller parts or spaces.

There may be several scales on the dial when the meter is used for different ranges. This is accomplished by switching shunts or multipliers as discussed later in this Section. The range of a meter means the highest value on that particular scale. Care must be taken, when reading these meters, to interpret the reading with relation to the selected range or scale only.

1.3 Example No. 1. The diagram shows the meter scale on a moving coil milliammeter which has a range of 0-150mA.



(i) What is the value of each small space between the main divisions?

(ii) What is the meter reading indicated by arrow A?

(iii) What is the meter reading indicated by arrow B?

Solution. (i) There are 5 small spaces between each main division of 25mA ; or 30 small spaces over the 150mA range. Therefore, each small division $= 150 \div 30 = 5$mA.

(ii) Arrow A is 4 small spaces past the 25mA reading. Each small space represents 5mA. Therefore, the meter reading $= 25 + (5 \times 4) =$ 45mA.

(iii) Arrow B is, by observation, about $2\frac{1}{4}$ small spaces past the 75mA reading. Each small space represents 5mA. Therefore, the meter reading $= 75 + (5 \times 2\frac{1}{4}) = 86.25$mA.

In practice, it is usually satisfactory to give meter readings to two (or at the most, three) significant figures. The meter reading for arrow B is, therefore, approximately 86mA or 86.3mA.

Answers: (i) 5mA ; (ii) 45mA ; (iii) 86.3mA.

EXERCISES

1. Referring to the diagram in Example No. 1, the meter reading indicated by -

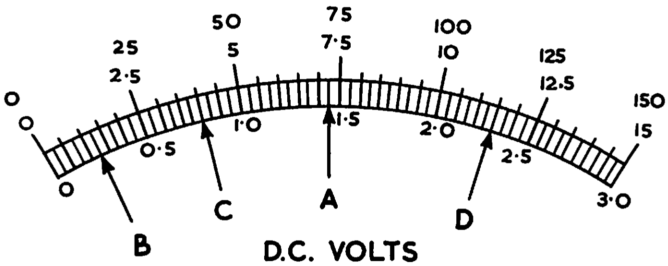
(i) Arrow C is .... mA

(ii) Arrow D is .... mA

(iii) Arrow E is .... mA

(iv) Arrow F is .... mA

1.4 Example No. 2. Find the reading indicated by arrow A for each scale of the dial shown in the diagram.



Solution. (i) On the 150V scale, there are 10 small spaces between each main division of 25V. Each small space, therefore, represents 2.5V. Arrow A is 9 small spaces past the 50V reading.

Therefore, the meter reading = 50 + (2.5 × 9) = 72.5V.

(ii) On the 15V scale, there are 10 small spaces between each main division of 2.5V. Each small space, therefore, represents 0.25V. Arrow A is 9 small spaces past the 5V reading.

Therefore, the meter reading = 5 + (0.25 × 9) = 7.25V.

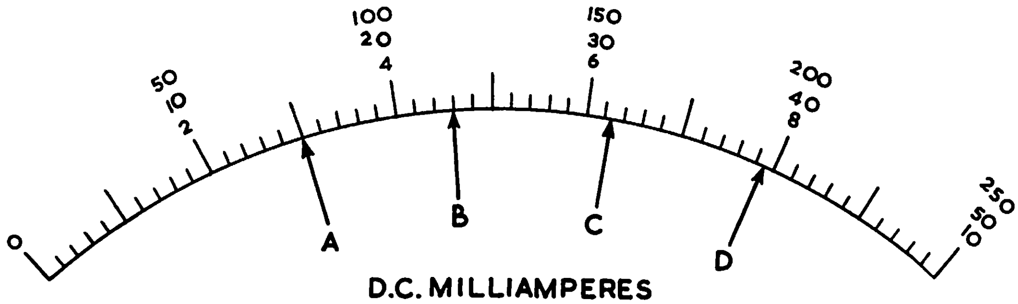
(iii) On the 3V scale, there are 10 small spaces between each main division of 0.5V. Each small space, therefore, represents 0.05V. Arrow A is 9 small spaces past the 1V reading.

Therefore, the meter reading = 1 + (0.05 x 9) = 1.45V.

Answers: (i) 72.5V ; (ii) 7.25V ; (iii) 1.45V.

EXERCISES

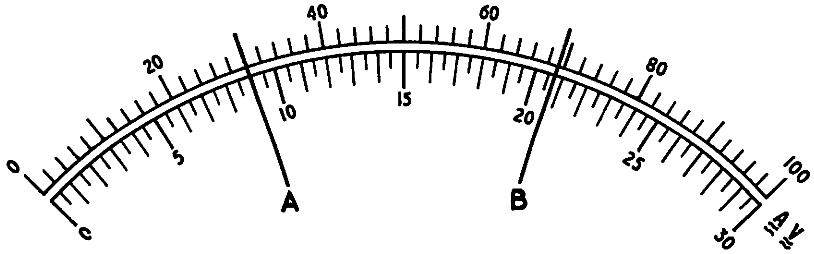
1. Referring to the diagram in Example No. 2, the meter reading indicated by -
   - (i) Arrow B on the 15V scale is .... V.
   - (ii) Arrow C on the 3V scale is .... V.
   - (iii) Arrow D on the 150V scale is .... V.
2. Referring to the following diagram, what is the meter reading for each scale, at the points indicated by the arrows?



Arrow	10mA scale	50mA scale	250mA scale
A			
B			
C			
D			

1.5 The diagram shows current and voltage scales used on the A.P.O. Multimeter No. 3. This instrument provides for D.C. measurements over the following ranges -

Current :	0-1mA ;	0-10mA ;	0-100mA ;	0-1A ;	0-10A.	
Voltage :	0-3V ;	0-10V ;	0-30V ;	0-100V ;	0-300V ;	0-1,000V.



The upper scale is used for current measurements, and any reading on this scale must be interpreted with relation to the range selected. For example, on the 1mA range, the main division marked 40 indicates a current of 0.4mA; on the 10A range, it indicates 4A.

Both scales are used for voltage measurements, and any reading must be similarly interpreted. For example, on the 3V range (lower scale), the main division marked 20 indicates a voltage of 2V; on the 1,000V range (upper scale), the main division marked 20 indicates 200V.

1.6 Summary. To read the value indicated by the pointer on a meter scale -

(i) Determine which of the several scales is to be used.

(ii) Note the value of the last main division passed by the pointer.

(iii) Determine the value of each small space, and how many small spaces past the last main division have been passed.

(iv) Add the small-space value to the main-division value to obtain the total reading.

EXERCISES

Referring to the diagram in para. 1.5 for the A.P.O. Multimeter No. 3 -

1. What is the value of a small-space division for each of the following ranges?

1mA	3V ....
10mA	10V ....
100mA	30V ....
1A	100V ....
10A	300V ....

2. What is the reading of pointer A for each of the following ranges?

1mA	3V ....
10mA	10V ....
100mA	30V ....
1A	100V ....
10A	1,000V ....

3. What is the reading of pointer B for each of the following ranges?

1mA	3V ....
1A	1,000V ....

2. MOVING COIL AMMETER.

2.1 A moving coil ammeter is used to measure the current in an electric circuit. It is connected in series with the circuit.

The value of current required to cause the pointer to swing over the entire scale indicates the sensitivity of the meter movement. This is called the full-scale deflection (F.S.D.) current. For example -

an 0-100mA meter requires 100mA for full-scale deflection;
an 0-500μA meter requires 0.5mA for full-scale deflection.

The more sensitive movement requires a smaller current to produce a given deflection.

To extend the current range, a resistance (called a shunt) is connected in parallel with the meter to divert all current in excess of the F.S.D. value. The meter scale is calibrated in terms of the circuit current.

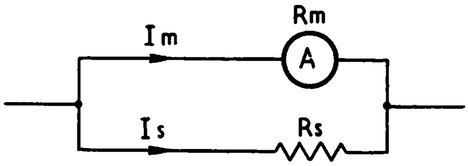
2.2 Formula to find value of shunt resistance. Applying the rule "the same voltage exists across each resistance in a parallel group" -

Voltage across meter $= I_m R_m$

Voltage across shunt $= I_s R_s$

Therefore .... $I_s R_s = I_m R_m$

$$R_s = \frac{I_m R_m}{I_s}$$



R_s and R_m = resistance (ohms) of shunt and meter respectively;

I_s and I_m = current through shunt and meter respectively. Any unit of current - A, mA or μA - can be used as long as both currents are expressed in the same unit.

As an exercise, develop the formula for R_s, applying the rule "the currents through two resistances in parallel are in the inverse ratio of the respective resistances."

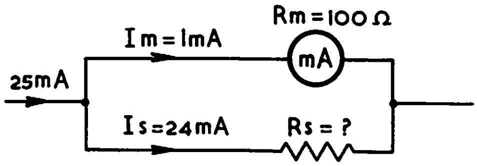
2.3 Example No. 3. A meter with an F.S.D. of 1mA has a resistance of 100 ohms. What shunt resistance is necessary to extend its range to 25mA?

Solution. When 25mA flows in the circuit, only 1mA flows through the meter, and the remainder (24mA) flows through the shunt.

$$R_s = \frac{I_m R_m}{I_s}$$

$$= \frac{1 \times 100}{24}$$

$$= 4.167 \text{ ohms.}$$



Answer: 4.17 ohms.

Note that the shunt current is 24 times the meter current; therefore, the shunt resistance is 1/24th of the meter resistance.

As an exercise, check this answer by an alternative method -

First find the P.D. across the meter when 1mA flows through it. This P.D. is the same as that across the shunt when 24mA flows through it. Then find the value of shunt resistance by applying Ohms Law to the two known values.

2.4 One way to find the resistance of a moving coil meter is to connect a resistance of known value, in parallel; and determine the division of current between the meter and the shunt.

Example No. 4. Calculate the resistance of an 0-1mA moving coil meter which has a resistance of 2.7 ohms connected in parallel, and which reads 0.6mA when connected in a circuit carrying 25mA?

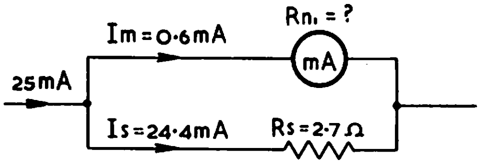
Solution. When 25mA flows in the circuit and 0.6mA through the meter, the remainder (24.4mA) flows through the shunt.

$$I_m R_m = I_s R_s$$

Therefore, $$R_m = \frac{I_s R_s}{I_m}$$

$$= \frac{24.4 \times 2.7}{0.6} = 109.8 \text{ ohms.}$$

Answer: 110 ohms.



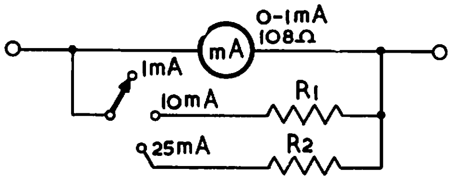
EXERCISES

1. Which is the more sensitive meter movement - one with an F.S.D. of 1mA or 5mA?

2. An 0-2mA meter has a resistance of 75 ohms. What value of shunt will permit it to measure currents up to 50mA?

3. A 50µA meter movement has a resistance of 2,000 ohms. What shunt resistance is required for a full-scale reading of 10mA?

4. The diagram shows an 0-1mA meter with a resistance of 108 ohms, connected so that the range can be extended to 10mA and 25mA by switching in R_1 and R_2 respectively. Calculate the value of each shunt.



5. A meter has 40 ohms resistance and reads full-scale deflection when connected across 40mV. How could you use this meter to read full-scale when connected in a circuit carrying 5mA?

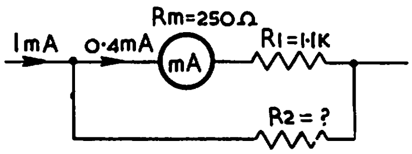
6. Calculate the resistance of an 0-10mA moving coil meter which is shunted with a resistance of 7 ohms and which reads its F.S.D. value when connected in a circuit carrying 25mA.

7. What is the resistance of an 0-5mA meter which requires a shunt resistance of 2.1 ohms to extend its range to 50mA?

8. An 0-5mA meter has its range extended to 10mA by a shunt resistor R_1 of 20 ohms. What value of resistance (R_2) must be connected in parallel with this combination to further extend the range to 5 amperes?

9. An 0-1mA meter has a shunt resistance of 1 ohm on the 100mA range. What value of shunt would be required for the 10mA range?

10. The diagram shows the simplified circuit of the A.P.O. Multimeter No. 3 when used on the 1mA range. The meter requires 0.4mA for F.S.D. and has a resistance of 250 ohms. Calculate the value of the shunt resistor R_2.



3. MOVING COIL VOLTMETER.

3.1 A voltmeter is used to measure the voltage across two points in a circuit. It is connected in parallel with the portion of the circuit being measured.

The moving coil voltmeter is basically an ammeter with a high resistance (called a multiplier) connected in series with the coil. This extra resistance limits the current to the value required for full-scale deflection when the full-scale voltage is applied across the voltmeter.

3.2 The sensitivity (in ohms/volt) of a moving coil meter movement can be found by taking the reciprocal of the full-scale current (in amperes).

$$\text{Sensitivity (ohms/volt)} = \frac{1}{\text{F.S.D. current (amperes)}}$$

For example, an 0–1mA meter movement has a sensitivity of 1,000 ohms per volt;
an 0–50μA meter movement has a sensitivity of 20,000 ohms per volt.

This indicates that, using an 0-1mA meter movement in a voltmeter, we need 1,000 ohms of meter resistance for every volt applied.

3.3 The sensitivity of a voltmeter can also be expressed as the ratio of its total resistance (R) to the voltage (E) required for full-scale deflection.

$$\text{Sensitivity (ohms/volt)} = \frac{R \text{ (ohms)}}{E \text{ (volts)}}$$

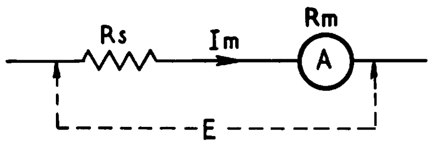
3.4 Formula to find value of series multiplier. In the following series circuit -

$$\text{Total Resistance} = R_s + R_m$$

$$\text{Also, total resistance} = \frac{E}{I_m}$$

$$\text{Therefore, } R_s + R_m = \frac{E}{I_m}$$

$$\text{and } R_s = \frac{E}{I_m} - R_m$$



where

R_s = resistance (ohms) of series multiplier
E = voltage (volts) to produce F.S.D.
I_m = current (amperes) to produce F.S.D.
R_m = resistance (ohms) of meter.

The resistance of the meter may be neglected when it is very much smaller than (say, less than 1% of) the resistance of the multiplier, and this formula then simplifies to the basic Ohms Law formula -

$$R_s = \frac{E}{I_m}$$

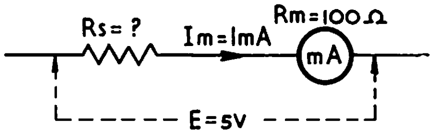
3.5 Example No. 5. What value of multiplier must be used to enable an 0-1mA meter with a resistance of 100 ohms to indicate 5 volts on full-scale deflection?

Solution.

$$R_s = \frac{E}{I_m} - R_m$$

$$= \frac{5}{0.001} - 100$$

$$= 5{,}000 - 100 = 4{,}900 \text{ ohms.}$$



Answer: 4,900 ohms.

Check this answer by an alternative method -

Sensitivity of the 0-1mA meter movement is 1,000 ohms per volt. Therefore, to measure 5 volts full-scale, we need 1,000 × 5 = 5,000 ohms in series. As the meter movement = 100 ohms, the multiplier = 5,000 - 100 = 4,900 ohms.

EXERCISES

1. What is the sensitivity in ohms per volt, of a voltmeter which uses a meter with an F.S.D. of 10mA ; 1mA ; 50µA?

2. What is the resistance of a voltmeter on the 50V range, if it uses a meter with a sensitivity of -

   (i) 100 ohms/volt ; (ii) 1,000 ohms/volt ; (iii) 20,000 ohms/volt?

3. What current is required for full-scale deflection of a voltmeter with a sensitivity of -

   (i) 100 ohms/volt ; (ii) 1,000 ohms/volt ; (iii) 20,000 ohms/volt?

4. What is the sensitivity in ohms per volt, of a voltmeter which indicates 50V full-scale, and has a resistance of 5,000 ohms ; 100 kilohms ; 1 megohm?

5. A 50µA meter movement has a resistance of 2,000 ohms. What multiplier resistance is required to measure 500V full-scale?

6. What voltage, connected across an 0-1mA meter with a resistance of 100 ohms, will cause full-scale deflection of the pointer?

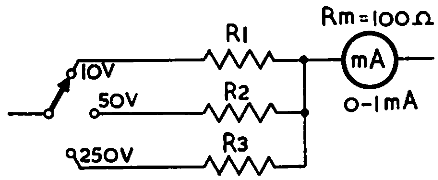
7. A voltmeter which reads 10V full-scale and has a resistance of 10,000 ohms, is to be extended to read 100V full-scale. What additional series resistance is needed, and what is the sensitivity in ohms/volt of the 100V meter?

8. A meter has a resistance of 40 ohms and reads F.S.D. when connected across 40mV. How could you use this meter to read full-scale when it is connected across 500V?

9. What value of multiplier must be connected in series with an 0-10V meter with a sensitivity of 20,000 ohms per volt, to extend the full-scale reading to 50V?

10. What value of series resistor is used with a voltmeter having a resistance of 15,000 ohms, in order to have its readings multiplied by 5?

11. The diagram shows an 0-1mA meter with a resistance of 100 ohms connected so that it can be used as a voltmeter with ranges of 10V, 50V and 250V, by switching in series R_1, R_2 and R_3 respectively. Calculate the value of each multiplier.



12. A voltmeter which indicates 100V full-scale and has a sensitivity of 10,000 ohms per volt is connected across a 50V battery. What is the current through the meter?

13. What is the resistance of an 0-100µA meter movement which uses a series multiplier of 29,000 ohms on the 3V range?

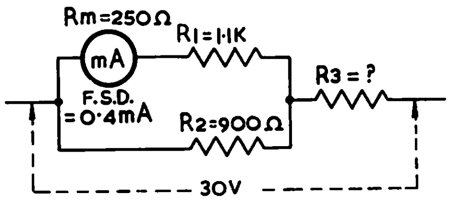
14. An 0-1mA meter movement uses a 9.9K series multiplier on the 10V range. What value of series multiplier would be required for the 3V range?

15. When a 10,000 ohm moving coil voltmeter is connected across 50V, the pointer deflects to the mid-scale position. Find the sensitivity of the voltmeter in ohms per volt.

16. A voltmeter uses a moving coil meter movement with a sensitivity of 20,000 ohms per volt. The total resistance of the moving coil and series multiplier is 10 megohms. What is the voltage across the voltmeter when the pointer reads mid-scale?

17. When a D.C. voltmeter with a sensitivity of 500 ohms per volt is connected across 25V, the current through the moving coil is 0.5mA. What is the voltage indicated by full-scale deflection of the meter?

18. The diagram shows the simplified circuit of the A.P.O. Multimeter No. 3 when used on the 30V range. The meter requires 0.4mA for F.S.D. and has a resistance of 250 ohms. Calculate the value of the multiplier R_3.



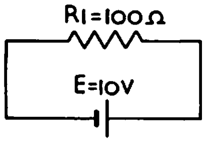
4. ACCURACY OF AMMETER READINGS.

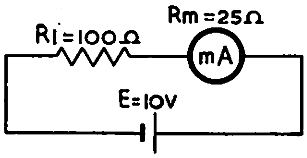
4.1 The resistance of an ammeter, including shunt, should be very much lower than the resistance of the circuit in which it is connected, so that it does not appreciably reduce the value of the current to be measured. When the ammeter resistance is comparable with (say, more than 1% of) the circuit resistance, its effect cannot be neglected and must be considered in the calculations.

4.2 Example No. 6. (i) A circuit with a resistance of 100 ohms is connected to a 10V battery. What is the current in the circuit?

(ii) What is the meter reading when an 0-100mA meter with a resistance of 25 ohms is used to measure this current?

Solution.





(i) Circuit current $= \frac{E}{R_1}$

$= \frac{10}{100}$ A

$= 100$mA

(ii) Meter reading $= \frac{E}{R_1 + R_m}$

$= \frac{10}{100 + 25} = \frac{10}{125}$ A

$= 80$mA

Answers: (i) 100mA ; (ii) 80mA.

(Note that the insertion of an ammeter reduces the current in the circuit; a comparatively high resistance ammeter appreciably reduces the current. In this example, the meter reading (80mA) does not accurately indicate the value of the current to be measured (100mA), and is 20% low.)

EXERCISES

1. In Example No. 6, what is the meter reading and the percentage error when the meter resistance is reduced to (i) 10 ohms; (ii) 1 ohm?

   (Note that the lower the meter resistance, the less effect it has on the circuit, and the more accurately it indicates the value of the current to be measured.)

2. An 0-400μA meter with a resistance of 250 ohms and an 0-1mA meter with a resistance of 100 ohms are used, in turn, to measure the current drawn by a 3.75K resistor connected to a 1.5V dry cell. Calculate the meter reading in each case.

   (Note that the readings differ for each meter. In the above problem, which meter gives the more accurate indication of the circuit current to be measured?)

3. The meters referred to in Exercise No. 2 are used, in turn, to measure the current drawn by a 37.5K resistor connected to a 15V battery. Calculate the reading in each case.

   (Note that both meters now read practically the same, because the resistance of each is much lower than the circuit resistance.)

4. An A.P.O. Multimeter No. 3 is used to measure the current drawn by a transistor amplifier which has a resistance of 900 ohms and is connected to a 9V battery. Calculate the meter reading on -

   (i) the 10mA range (meter resistance 86.4 ohms);
   (ii) the 100mA range (meter resistance 1.45 ohms).

   (Note that the readings differ for each range. In the above problem, which range gives the more accurate indication of the circuit current to be measured?)

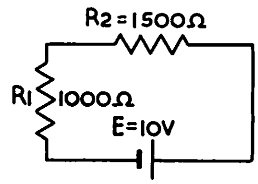
5. ACCURACY OF VOLTMETER READINGS.

5.1 The resistance of a voltmeter (meter plus multiplier) should be very much higher than the resistance of that part of the circuit across which it is connected. The voltmeter then draws only a small current, which does not appreciably alter the voltages in the circuit. When the voltmeter resistance is comparable with (say, less than 100 times) the resistance of that part across which it is connected, its effect cannot be neglected and must be considered in the calculations.

5.2 Example No. 7. (i) Two resistors, R_1 = 1,000 ohms and R_2 = 1,500 ohms, are connected in series with a 10V battery. What is the P.D. across R_1?

(ii) What is the meter reading when an 0-10V meter with a resistance (R_m) of 1,000 ohms (sensitivity 100 ohms/volt) is used to measure this P.D.?

Solution. (As an exercise, set out this problem in detail, using formulas.)

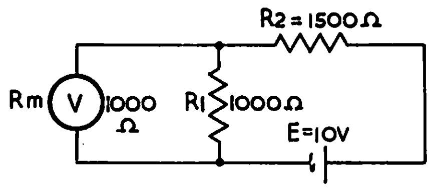


(i) The P.D's across R1 and R2 are in the ratio of –

1,000 : 1,500 or 2 : 3.

The P.D. across $R_1 = \frac{2}{2+3} \times \frac{10}{1} = 4V.$

(ii) Joint resistance of R1 and Rm in parallel = 500 ohms.



The P.D's in this circuit are in the ratio of –
500 : 1,500 or 1 : 3.

The P.D. across R1, which is also the meter reading (R_m)

$$= \frac{1}{1+3} \times \frac{10}{1} = 2.5V.$$

Answers: (i) 4V ; (ii) 2.5V.

(Note that the connection of a voltmeter reduces the voltage across that part of the circuit; a comparatively low resistance voltmeter appreciably reduces the voltage. In this example, the meter reading (2.5V) across R1 does not accurately indicate the value of the voltage to be measured (4V) and is 37.5% low. Note also that the voltage across R2 has risen from 6V to 7.5V.)

EXERCISES

1. In Example No. 7, what is the meter and the percentage error when the 0-10V meter has a sensitivity of (i) 1,000 ohms per volt, (ii) 10,000 ohms per volt?

   (Note that the higher the meter resistance, the less effect it has on the circuit, and the more accurately it indicates the value of the voltage to be measured.)

2. A 1,500 ohm relay and a 1K resistor are connected in series with a 50V battery. An 0-30V and an 0-75V meter, each with a sensitivity of 100 ohms/volt, are used in turn, to measure the voltage across the relay. Calculate each meter reading.

   (Note that the readings differ for each meter. In the above problem, which meter gives the more accurate indication of the voltage to be measured?

3. The meters referred to in Exercise No. 2 are used, in turn, to measure the P.D. across a 150 ohm resistor connected in series with a 100 ohm resistor and a 50V battery. Calculate the meter reading in each case.

   (Note that both meters now read practically the same, because the resistance of each is much higher than the resistance of 150 ohms across which they are connected.)

4. (i) Two resistors, R_1 = 15K and R_2 = 85K, are connected in series with a 100V D.C. supply. What is the P.D. across R_1?

   (ii) What is the meter reading when an A.P.O. Multimeter No. 3 with a sensitivity of 1,000 ohms per volt on all voltage ranges, is used to measure this P.D. on the 0-100V, 0-30V, 0-10V and 0-3V range?

   (Note that the readings differ appreciably for each range, because of the different meter resistances. Why does the meter not exceed full-scale deflection on the 3V range?)

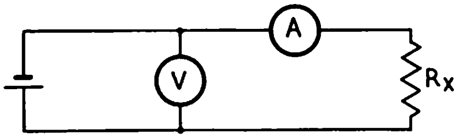
2. MEASUREMENT OF RESISTANCE

Reference: APPLIED ELECTRICITY 1, "D.C. Measurements", Sections 4 and 5.

1. AMMETER-VOLTMETER METHOD.

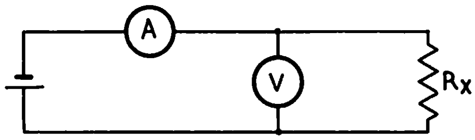
1.1 A simple way to determine the value of an unknown resistance (R_X) is to pass a current through it and measure the current (I) with an ammeter, and the voltage drop (E) with a voltmeter. The resistance is then given by Ohms Law ($R_X = E/I$).

1.2 Either of two circuit arrangements can be used, the accuracy depending on the ratio of the meter resistances to the resistance of the circuit under test.



(i)

Arrangement (i) is preferred when the voltmeter resistance is comparable with R_X. The ammeter reads the current through R_X correctly, but the voltmeter reads the sum of the voltages across the ammeter and R_X. The ammeter resistance must be very low compared to R_X, so that the voltage reading more accurately indicates the voltage across R_X.



(ii)

Arrangement (ii) is preferred when the ammeter resistance is comparable with R_X. The voltmeter reads the voltage across R_X correctly, but the ammeter reads the sum of the currents through the voltmeter and R_X. The voltmeter resistance must be very high compared to R_X, so that the current reading more accurately indicates the current through R_X.

When the voltmeter has a very high resistance and the ammeter has a very low resistance compared to R_X, either arrangement gives sufficient accuracy for most purposes. In problems where the resistances of the meters are not stated, they can be assumed to have negligible effect on the circuit.

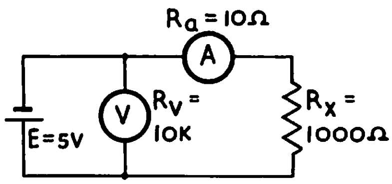
1.3 Example No. 1. A 1,000 ohm resistor is to be checked by the ammeter-voltmeter method using the arrangements in (i) and (ii) above. The battery voltage is 5V. The meters used are an 0-10mA meter with a resistance of 10 ohms, and an 0-10V meter with a resistance of 10,000 ohms (sensitivity of 1,000 ohms per volt). What is the resistance value calculated from the meter readings for each arrangement? Which arrangement gives the more accurate indication of the value of R_X?

Solution. (The values of the meter readings are left in fraction form to simplify the calculations.)

(i) Voltmeter reading (E_m) = 5V

Ammeter reading (I_m) $= \dfrac{E}{R_a + R_x} = \dfrac{5}{1{,}010}$ A

Calculated value of $R_x = \dfrac{E_m}{I_m}$ = 1,010 ohms.

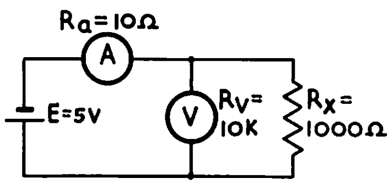


(ii) Joint resistance (R_j) of R_V and R_X in parallel = 909Ω ; and total resistance (R_T) of circuit = 919Ω

Voltmeter reading (E_m) $= \dfrac{R_j(E)}{R_a + R_j} = \dfrac{909 \times 5}{919}$ V

Ammeter reading (I_m) $= \dfrac{E}{R_T} = \dfrac{5}{919}$ A

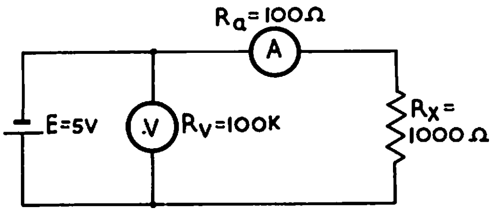
Calculated value of $R_X = \dfrac{E_m}{I_m}$ = 909 ohms.



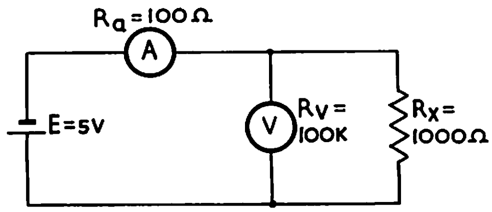
Answers : (i) 1,010 ohms ; (ii) 909 ohms.

Note that (i) is 1% high and (ii) is 10% low. As the voltmeter resistance is comparable with R_X (ratio of 10 : 1), the meter readings in arrangement (i) give a more accurate indication of the value of R_X.

1.4 Repeat the calculations in Example No. 1, when the 0-10mA meter has a resistance of 100 ohms, and the 0-10V meter has a resistance of 100,000 ohms.



(i)



(ii)

The answers are (i) 1,100 ohms and (ii) 990 ohms.

Note that (i) is 10% high and (ii) is 1% low. As the ammeter resistance is comparable with R_X (ratio of 1 : 10), the meter readings in arrangement (ii) give the more accurate indication of the value of R_X.

1.5 Repeat the calculations in Examples Nos. 1 and 2, comparing the meters of different resistances when the battery voltage is increased to 10V.

Note that the calculated values of resistance are unchanged and do not depend on the battery voltage, provided the resistance of the meters are unchanged. However, in practice, a higher voltage may allow the ammeter to be switched to a higher range (which has less resistance) and/or the voltmeter to be switched to a higher range (which has more resistance), thus giving better accuracy in the calculations.

EXERCISES

1. A transistor amplifier operating from a 9V battery draws a current of 10mA. What is the resistance of the amplifier?

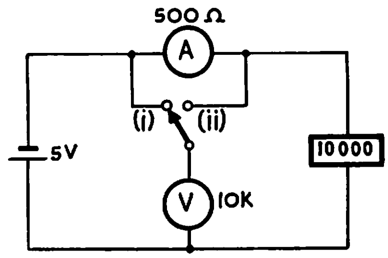
2. A voltmeter across the terminals of a telephone connected to a telephone line, shows a reading of 5.85V; and a milliammeter in series with the line reads 75mA. What is the resistance of the telephone?

3. It is desired to check the resistance of a 10,000 ohm relay using the ammeter-voltmeter method, as shown in the diagram. Two A.P.O. Multimeters No. 3 are used on the following ranges -

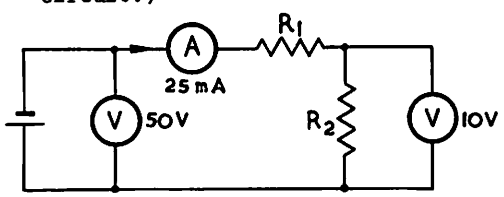
   0-1mA (approx. resistance 500 ohms),
   0-10V (resistance 10,000 ohms).

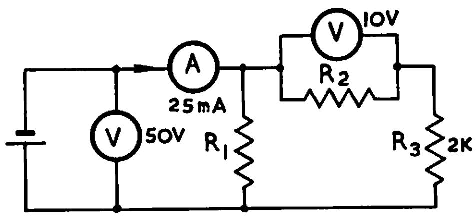
   (i) From the calculated meter readings, what is the calculated value of the relay resistance for each position of the switch?

   (ii) Which position gives the more accurate indication of the relay resistance?



4 and 5. Calculate the resistance of R_1 from the meter readings in each of the following diagrams. (Assume that the meters have negligible effect on the circuit.)





2. SERIES VOLTMETER METHOD.

2.1 To measure resistance by the series voltmeter method, connect a voltmeter of known resistance (R_m) in series with a known voltage (E_b) and the unknown resistance (R_x). Observe the voltmeter reading (E_m), and subtract E_m from E_b to find the voltage (E_x) across the unknown resistance. Then calculate the value of R_x as shown in para. 2.2.

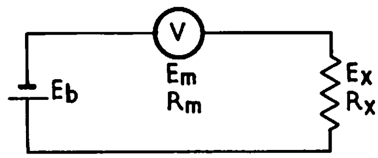
2.2 Formula to find value of unknown resistance. The voltages across resistances in series are directly proportional to the respective resistances, that is -

$$\frac{E_x}{E_m} = \frac{R_x}{R_m}$$

Transposing, $R_x = \frac{R_m E_x}{E_m}$

where E_x (voltage across R_x) $= E_b - E_m$

Therefore, $R_x = \frac{R_m(E_b - E_m)}{E_m}$



(Note: In A.E.1, the symbols V_1 and V_2 are used for E_b and E_m respectively.)

2.3 Example No. 2. Using the series voltmeter method, calculate the insulation resistance in megohms of a telephone line when the testing voltage is 80V, the voltmeter resistance is 200K and the voltmeter reading is 8V.

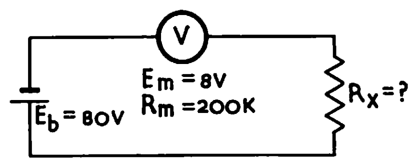
Solution.

$$R_x = \frac{R_m(E_b - E_m)}{E_m}$$

$$= \frac{200{,}000\ (80 - 8)}{8}$$

$$= 1{,}800{,}000 \text{ ohms}$$

Answer : 1.8 megohms.



EXERCISES

1. In Example No. 2, what is the insulation resistance when the voltmeter reads -

   (i) 2V : (ii) 40V : (iii) 75V?

   (Note that higher voltmeter readings indicate lower values of insulation resistance.)

2. A voltmeter on the 5V range reads 2.25 volts when connected in series with a 4.5V battery and an unknown resistance. What is the value of the resistance, assuming the voltmeter has a sensitivity of -

   (i) 1,000 ohms per volt; (ii) 20,000 ohms per volt?

   (Note that, for the same deflection, the higher sensitivity meter enables higher resistance values to be measured.)

3. A multi-range voltmeter with a sensitivity of 1,000 ohms per volt, is set on the 300V range and connected in series with a resistance R_x and a 220V power supply.

   (i) What is the value of R_x, when the meter reads 110V?

   (ii) The voltmeter is then switched, in turn, to the lower ranges. What is the reading when switched to -

   the 100V range ....

   the 30V range ....

   the 3V range ....?

   (Why does the voltmeter reading progressively decrease on each lower range?)

3. VOLTAGE COMPARISON METHOD.

3.1 To measure resistance by the voltage comparison method, connect a known or standard resistance in series with the unknown resistance. Measure the P.D. across each resistance in turn, using the same meter without change of scale.

The respective P.Ds. have the same ratio as the resistance values. For example, referring to Example No. 3, if the voltmeter reading E2 is twice that of E1, then R2 has twice the resistance of R1.

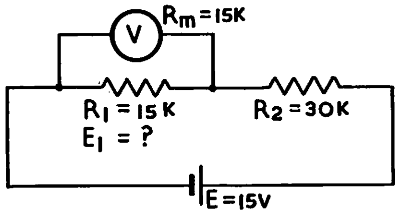
$$\boxed{\frac{E_1}{E_2} = \frac{R_1}{R_2}}$$

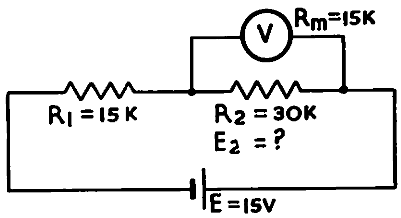
It is interesting to note that, provided the applied voltage is kept constant and the same voltmeter is used, the ratio of the voltmeter readings is the same irrespective of the sensitivity of the voltmeter used.

IMPORTANT. This method applies only to two resistances in series.

3.2 Example No. 3. A 15K and a 30K resistor are connected in series with a 15V D.C. supply. An 0-15V meter with a resistance of 15,000 ohms is connected in turn across each resistor. What are the voltage readings across (i) the 15K resistor, and (ii) the 30K resistor?

Solution.





(i) Calculate the joint resistance of the 15K meter and the 15K resistor in parallel –

$$R_j = \frac{R_m \times R_1}{R_m + R_1} \quad \text{................} \quad 7.5K$$

Calculate the P.D. across this combination, which equals the meter reading –

$$E = \frac{R_j(E)}{R_j + R_2} \quad \text{..............} \quad 3V$$

(ii) Calculate the joint resistance of the 15K meter and the 30K resistor in parallel –

$$R_j = \frac{R_m \times R_2}{R_m + R_2} \quad \text{................} \quad 10K$$

Calculate the P.D. across this combination, which equals the meter reading –

$$E = \frac{R_j(E)}{R_1 + R_j} \quad \text{...............} \quad 6V$$

Answer: (i) 3 volts; (ii) 6 volts.

Note. The ratio of the voltage readings is the same as the ratio of the respective resistances.

EXERCISES

1. In Example No. 3, what are the voltage readings across (i) the 15K resistor, and (ii) the 30K resistor, when the voltmeter has a resistance of 10,000 ohms?

   (Note that the voltage readings are lower than those obtained when a higher resistance voltmeter is used. (True values are 5V and 10V respectively.) However, they still have the same ratio as the ratio of the resistances.)

2. Relay A with a resistance of 125 ohms is connected in series with relay B and a battery. An 0-50V meter reads 14V when connected across relay A, and 28V across relay B. What is the resistance of relay B?

3. A battery is connected via a 1K resistor to a telephone line which is short circuited at the distant end. A voltmeter reads 25V when connected across the resistor, and 9.5V when connected across the line. What is the line resistance?

4. A 1,000 ohm relay and a 1,500 ohm resistor are connected in series with a 50V battery. An 0-30V meter of low sensitivity reads 10V when connected across the relay.

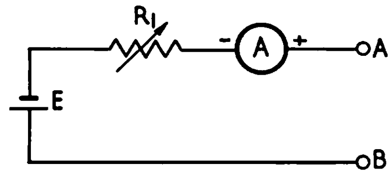
   (i) What does the voltmeter read when connected across the resistor?

   (ii) What is the voltmeter sensitivity in ohms per volt?

4. THE OHMMETER.

4.1 A simple ohmmeter consists of a specially calibrated ammeter in series with a variable resistor (R_1) and a cell or battery (E), as shown in the diagram. With an open circuit (infinite resistance) between terminals A and B, no current flows in the circuit and the meter reading is zero.

Adjust resistor R_1 so that the meter reads its F.S.D. value when a short-circuit (zero resistance) is applied between the terminals. (In practice, a fixed resistor and a variable resistor are connected in series; and the variable resistor is adjusted to compensate for small variations in battery voltage.)



Remove the short-circuit and connect the resistance of unknown value (R_x) between the terminals. The meter deflects to a value less than its F.S.D. value, and its scale is calibrated in ohms instead of amperes so that the value of R_x can be read direct.

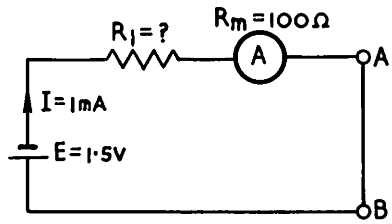
When R_x equals the resistance of the ohmmeter (that is, total resistance of ammeter, resistor R_1 and battery in series), the meter reads mid-scale (half full-scale) deflection.

4.2 Example No. 4. An 0-1mA meter (resistance 100 ohms) is used as an ohmmeter in conjunction with a series resistor and a 1.5V dry cell with negligible internal resistance. Calculate -

(i) the value of the series resistor (R1);
(ii) the external resistance (R_x) when the meter reads 0.25mA;
(iii) the P.D. (E_x) across the external resistance.

Solution. (i) For the meter to read 1mA F.S.D. when the ohmmeter terminals are short-circuited, the total resistance (R_o) of the ohmmeter circuit must be -

$$R_o = \frac{E}{I} = 1{,}500 \text{ ohms.}$$

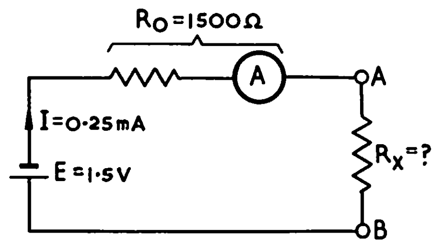


To find the value of the series resistor (R_1), subtract the meter resistance (R_m) from the ohmmeter resistance (R_o) -

$$R_1 = R_o - R_m = \underline{1{,}400 \text{ ohms.}}$$

(ii) When the current in the circuit is reduced to 0.25mA due to R_x, the total resistance (R_T) of th series circuit is -

$$R_T = \frac{E}{I} = 6{,}000 \text{ ohms}$$



To find the value of the unknown resistance (R_x), subtract the ohmmeter resistance (R_o) from the total resistance (R_T) -

$$R_x = R_T - R_o = \underline{4{,}500 \text{ ohms.}}$$

(iii) $$E_x = \frac{R_x(E)}{R_T} = \underline{1.125V.}$$

Answers: (i) 1,400 ohms ; (ii) 4,500 ohms ; (iii) 1.125V.

Note that portion of the battery voltage is dropped across R_x. When using ohmmeters, check that this voltage does not exceed the rating of the component under test. This applies particularly to transistors and other components (for example, electrolytic capacitors) in transistorised equipment.

When testing some components (for example, semiconductors), the ohmmeter readings must be interpreted with regard to the applied polarity. Generally, in multimeters, the negative terminal is of positive polarity when used on the ohmmeter ranges.

4.3 The ohmmeter scale is non-linear and is cramped at low current (high resistance) values. (See Exercise No. 1 below.)

To measure higher values of resistance -

(i) use a higher voltage battery. (See Exercises Nos. 2 and 3.) This requires a higher value of series resistance R_1 to limit the meter current to its F.S.D. value; and the P.D. across R_x is also higher.

(ii) use a more sensitive meter. (See Exercise No. 4.)

EXERCISES

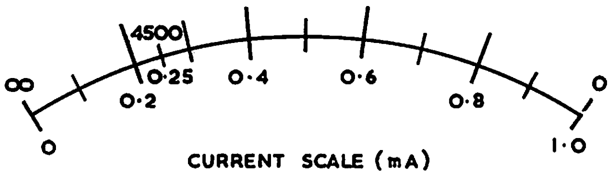
1. From the information given in Example No. 4 -

(i) Calculate the values of R_x which produce currents of 0.1mA, 0.2mA, etc., in 0.1mA steps up to 1mA. Enter the values in the table.

(ii) On the following scale, mark the values of R_x as an "ohms scale" corresponding to the current values.

Meter current (mA).	Value of R_x (ohms).
0.1	
0.2	
0.3	
0.4	
0.5	
0.6	
0.7	
0.8	
0.9	
1.0	

RESISTANCE SCALE (OHMS)



CURRENT SCALE (mA)

The mathematical symbol for infinity is ∞, and this indicates the open circuit (or infinite resistance) position on the scale.

2. An 0-1mA meter (resistance 100 ohms) is used as an ohmmeter in conjunction with a series resistor and a 15V battery with negligible internal resistance. Calculate -

(i) the value of the series resistor (R_1).
(ii) the external resistance (R_x) when the meter reads 0.25mA.
(iii) the P.D. across R_x.

(Compare your answers with those given in Example No. 4.)

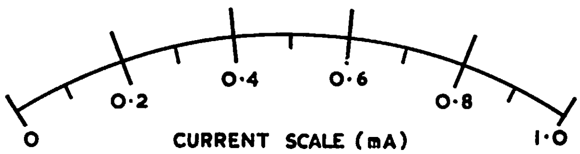
3. From the information given in Exercise No. 2 -

(i) Calculate the values of R_x which produce the current values indicated in the table.

(ii) On the following scale, mark the values of R_x as an "ohms scale" corresponding to the current values.

Meter current (mA).	Value of R_x (ohms).
0	
0.2	
0.4	
0.6	
0.8	
1.0	

RESISTANCE SCALE (OHMS)



CURRENT SCALE (mA)

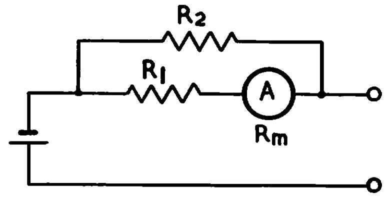
(Compare the scale with that in Exercise No. 1.)

4. An 0-100μA meter (resistance 1,000 ohms) is used as an ohmmeter in conjunction with a series resistor and a 1.5V dry cell with negligible internal resistance. Calculate -

(i) the value of the series resistor (R_1).
(ii) the external resistance (R_x) when the meter reads 25μA.
(iii) the P.D. across R_x.

(Compare your answers with those of Example No. 4 and Exercise No. 2 above.)

4.4 Using ohmmeter for low resistances. We can extend the range of an ohmmeter to read high values of resistance by using a higher battery voltage and/or a meter with higher sensitivity. Conversely, to read lower resistances, reduce the battery voltage; or where this is not practicable, reduce the resistance of the ohmmeter by a shunt resistance as shown in the diagram.



As before, the series resistor R_1 limits the current through the meter to its F.S.D. value when a short-circuit is applied across the terminals.

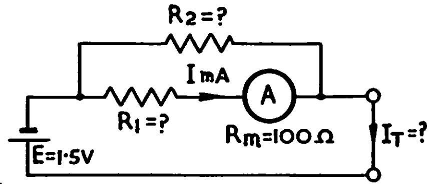
The shunt resistor R_2 reduces the overall resistance of the ohmmeter to a value which corresponds to mid-scale deflection on the ohms scale.

4.5 Example No. 5. In the above ohmmeter circuit, an 0-1mA moving coil meter with a resistance of 100 ohms is used in conjunction with a 1.5V dry cell. If mid-scale deflection on the meter is marked 150 ohms, calculate the values of -

(i) the series resistor R_1;

(ii) the shunt resistor R_2;

(iii) the current through a wire of negligible resistance connected across the terminals.



Solution. (i) R_1 is calculated as shown in Example No. 4(i), and equals 1,400 ohms.

(ii) Using the formula for parallel resistances, calculate the value of R_2 which, when connected in parallel with 1,500 ohms (the value of $R_1 + R_m$), gives a total ohmmeter resistance (R_o) of 150 ohms.

$$R_2 = \frac{(R_1 + R_m) \times R_o}{(R_1 + R_m) - R_o} = \underline{166.7 \text{ ohms.}}$$

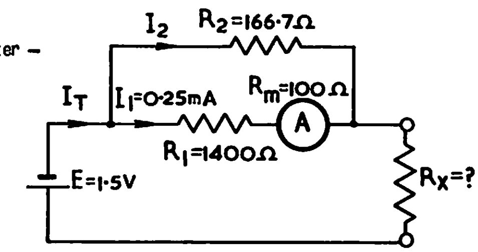
(iii) $$I_T = \frac{E}{R_o} = \underline{10\text{mA}}.$$

Answers : (i) 1,400 ohms; (ii) 166.7 ohms; (iii) 10mA.

Note that the total current in the circuit (10mA) is higher than the meter current (1mA). When using this type of ohmmeter to measure low values of resistance, it is important to check that this higher current does not damage the component under test.

4.6 Example No. 6. Using the ohmmeter circuit as in Example No. 5, calculate the value of the external resistance R_x, when the current through the meter is 0.25mA.

Solution. One way to work out this problem is as follows -



Calculate the current through the shunt resistance of the ohmmeter -

$$I_2 = \frac{I_1(R_1 + R_m)}{R_2} = 2.25\text{mA}$$

Calculate the total current flowing through the circuit -

$$I_T = I_1 + I_2 = 2.5\text{mA}$$

Calculate the total resistance of the circuit: $$R_T = \frac{E_T}{I_T} = 600 \text{ ohms.}$$

Calculate the resistance of the ohmmeter : $$R_o = \frac{R_2(R_1 + R_m)}{R_2 + R_1 + R_m} = 150 \text{ ohms.}$$

Calculate the value of the unknown resistance : $$R_x = R_T - R_o = 450 \text{ ohms.}$$

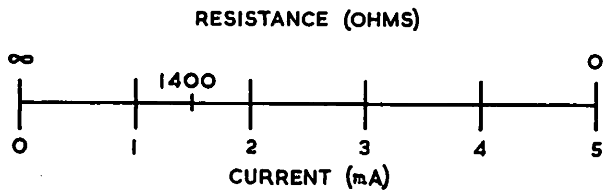
Answer : 450 ohms.

1. A meter with an F.S.D. of 500μA and a resistance of 200 ohms is used in an ohmmeter circuit to read 3K mid-scale. The voltage supply is a 1.5V dry cell. What is the value of the series resistor required?

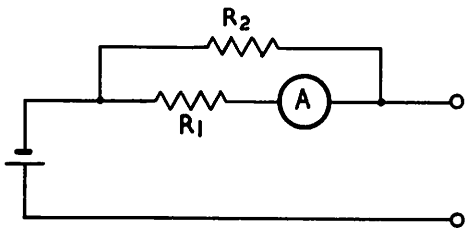
2. An 0-5mA meter, a 3V battery and a series resistor are used to form an ohmmeter.

   (i) On the straight-line meter scale shown below, mark in the values of the external resistance which produce currents of 1mA, 2mA, 3mA and 4mA in the circuit.

   (ii) Calculate the current in the circuit, when the external resistance is 1,400 ohms.



3. An ohmmeter consists of the following - 0-1mA meter, with a resistance of 80 ohms, a 1.5V dry cell, a limiting resistor R_1 and a shunt resistor R_2 connected as shown in the diagram. If half F.S.D. on the meter scale is marked 500 ohms, calculate the values of the two resistors.



4. In the ohmmeter circuit shown in para. 4.3, an 0-1mA (resistance 100 ohms) is used in conjunction with a 1.5V battery. If mid-scale deflection on the "low ohms" scale is marked 10 ohms, calculate the value of -

   (i) the series resistor R_1;

   (ii) the shunt resistor R_2;

   (iii) the current through a wire of negligible resistance connected across the terminals.

   (iv) the external resistance R_x, when the current through the meter is 0.25mA.

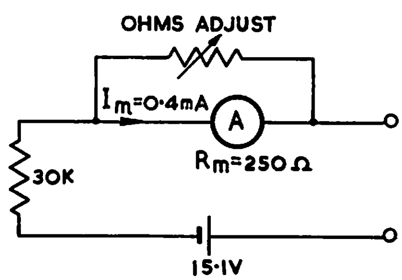
   (Compare these answers with those given in Examples Nos. 5 and 6. Note the relatively high current through the external resistance on the "low ohms" scale).

5. The diagram shows a simplified circuit of the A.P.O. Multimeter No. 3 on the "Ohms × 1000" range.

   (i) Assuming the battery voltage is 15.1V, to what value must the "Ohms Adjust" resistance be set to produce an F.S.D. of 400μA on the meter?

   (ii) What value of resistance connected across the terminals will produce mid-scale deflection on the meter?

   (iii) What is the P.D. across a 75.5K resistor connected across the terminals?



(Note the relatively high P.D. across the external resistance on the "high ohms" scale.)

5. THE WHEATSTONE BRIDGE.

5.1 The Wheatstone Bridge is used for accurate measurement of an unknown resistance. Four resistances A, B, R and X are arranged in two parallel branches as shown in the diagram. A, B, and R are variable known resistances, and X is the unknown resistance.

The variable resistances are adjusted until the galvanometer shows zero deflection. The bridge is then "balanced" and the galvanometer is connected across points of equal potential. (In practice, the "ratio arms" A and B are usually fixed at some convenient ratio and R is adjusted for balance.) When the bridge is balanced, a current I_1 flows through A and X, and I_2 through B and R. The voltages across A and B are equal, and the voltages across X and R are equal.

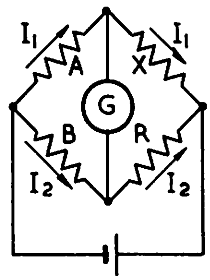
Therefore .. $I_1A = I_2B$

and $I_1X = I_2R$

Divide one equation by the other ... $\frac{I_1X}{I_1A} = \frac{I_2R}{I_2B}$

Cancel out the currents I_1 and I_2 ... $\frac{X}{A} = \frac{R}{B}$

or $BX = AR$



Note that when a bridge circuit is balanced, the products of the resistances in the opposite arms are equal. If you remember this, you will be able to develop the formula to find the value of any arm in a balanced bridge, provided the values of the other three arms are known. Note also that X = R, when the ratio arms have equal resistance (ratio of 1 : 1).

5.2 Example No. 7. Using the Wheatstone Bridge arrangement shown in the above diagram, the following readings were obtained at balance -

A = 10 ohms ; B = 100 ohms ; R = 55.4 ohms.

Find the value of the unknown resistance.

Solution. At balance .... $BX = AR$

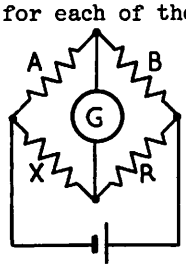
Therefore .... $X = \frac{AR}{B}$

$= \frac{10 \times 55.4}{100} = 5.54$ ohms.

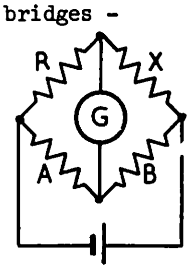
Answer : 5.54 ohms.

5.3 In some problems, the ratio arms may be connected directly across the battery or the arms of the bridge may be denoted by other symbols. In these cases, you may have to determine the formula for the particular problem before substituting the given values.

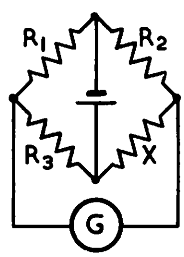
As an exercise, see if you can derive the formula for X in terms of the other three arms for each of the following bridges -



(i) $X = \frac{AR}{B}$



(ii) $X = \frac{BR}{A}$

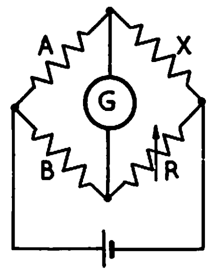


(iii) $X = \frac{R_2R_3}{R_1}$

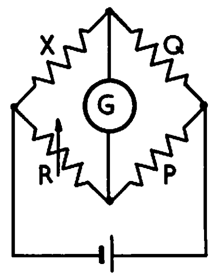
EXERCISES

(The readings in Problems 1 - 26 indicate the resistance in ohms in the arms of a Wheatstone Bridge when balanced. Find the value of the unknown resistance for each problem.)

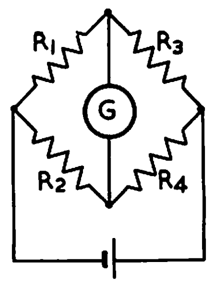
Problem	A	B	R	X
1	1,000	10,000	84.3	?
2	10,000	1,000	952.7	?
3	100	10	60.4	?
4	10	1	175.3	?
5	1	10	128.9	?
6	5,000	10,000	823.7	?
7	100	1,000	46.5	?
8	1,000	100	9.16	?



Problem	X	R	Q	P
9	?	32.5	1	100
10	?	18.24	10	1,000
11	?	1,175.0	1,000	1
12	?	1,175.0	1	1,000
13	?	100.0	100	100
14	?	1,000.0	10	100
15	?	50.0	1,000	10
16	?	18.2	100	1



Problem	R_1	R_2	R_3	R_4
17	100	?	1,000	10
18	?	15,000	2,500	6,000
19	870	430	?	2,150
20	?	1,000	87.5	2,500
21	2.5	8.5	4.8	?
22	1,000	?	10	15.25
23	240	16.8	45	?
24	102.5	?	1,000	102.5
25	?	810	352	648
26	316.8	873.2	?	151.6

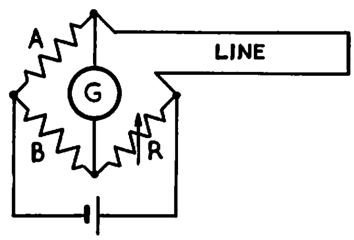


27. The Wheatstone Bridge is used to find the resistance of a telephone line, as shown in the diagram.

    Calculate the resistance of the line, when, at balance -

    (i) A : B :: 1 : 10, and R = 3,315 ohms;

    (ii) B : B ::10 : 1, and R = 33 ohms.



3. LOGARITHMS AND THE DECIBEL

Reference : Course Paper "Transmission Units". Sections 1 to 5.

1. COMMON LOGARITHMS.

1.1 Common logarithms are based on the principle that any number can be expressed as a power of 10. Calculations involving multiplication, division, raising to powers and extraction of roots can often be done more quickly by using logarithms than by direct arithmetical calculation. Of particular interest in telecom work is the application of logarithms in the calculation of electric power, voltage and current ratios.

This Section gives a basic understanding of logarithms and introduces problems on the decibel - the unit in which the above electrical ratios are expressed.

1.2 Powers of Ten. In Section 6, "Powers of Ten" of Telecom Mathematics 1, it was shown that numbers are often expressed as powers of ten to simplify the writing as well as the calculations involving them. In the expression -

$$1{,}000{,}000 = 10^6$$

the index 6 indicates that the "base" number 10 is to be raised to the sixth power.

The index 6 is called the logarithm (abbreviated to log) to the base of 10 of 1,000,000.

$$\overbrace{1{,}000{,}000}^{\text{NUMBER}} = \overset{\text{BASE}}{10}{}^{\overset{\text{LOG}}{6}}$$

Expressed in other words - "the log, to the base 10, of 1,000,000 is 6".

In mathematical terms, $\log_{10} 1{,}000{,}000 = 6$.

In common logarithms, the base is always 10, and is generally omitted in the above mathematical expression, which simplifies to -

$$\log 1{,}000{,}000 \text{ (or } \log 10^6) = 6.00$$

Similarly, $\log 1{,}000 \text{ (or } \log 10^3) = 3.00$

$$\log 100 \text{ (or } \log 10^2) = 2.00$$

$$\log 10 \text{ (or } \log 10^1) = 1.00$$

$$\log 1 \text{ (or } \log 10^0) = 0.00$$

1.3 Numbers between 1 and 10. Any number between 1 (which is $10^{0.00}$) and 10 (which is $10^{1.00}$) can be expressed as 10 raised to some power between 0 and 1. These powers have been calculated and listed in table form. The numbers from 1 to 10 have been shown to one decimal place, and for simplicity, the decimal point has been removed. The number 1.1 appears as 11 and the number 1.2 appears as 12 and so on to 9.9 which appears as 99. The log of the required number can then be read directly from these tables. A simplified set of "two figure" logarithm tables is shown at the end of this book.

Example No. 1. *To find the log of the number 2.0, read down the "Number" column of the Logarithm Tables to 20. Read across to the right to find 0.30 in the "Log" column.*

Therefore, $\log 2 = 0.30$

Similarly, $\log 3.7 = 0.57$

Example No. 2. *To find the log of a number which has more than two significant figures, first correct the number to two significant figures. Therefore:-*

$$\log 5.867 = 0.77$$

The degree of accuracy provided by these tables is satisfactory for our problems on electrical ratios and the decibel; but for greater accuracy, when logarithms are used to solve mathematical calculations, "four-figure" or "five-figure" tables can be used.

1.4 The log of a number consists of two parts - characteristic and mantissa.

The characteristic is the number before the decimal point and its value is determined by inspection. For numbers between:-

1 and 10,	the characteristic is 0;
10 and 100,	the characteristic is 1;
100 and 1,000	the characteristic is 2;
1,000 and 10,000	the characteristic is 3; and so on.

The characteristic of a whole number is always one less than the number of digits before the decimal point.

The mantissa is the decimal fraction of the logarithm, and this value is found from the tables. Note that, for powers of 10 only (for example, 10, 100, 1,000, etc.), the mantissa is always zero.

1.5 Numbers greater than 10. The same tables can be used to find the mantissa portion of the log of numbers greater than 10; but we must remember to add the appropriate value of characteristic to the mantissa.

Example No. 3. *To find the log of the number 20:-*

(i) *find the mantissa by reading down the "Number" column of the Logarithm Tables to 20 and across to 0.30 in the "Log" column.*

find the characteristic by inspection; for numbers between 10 and 100 the characteristic is 1.

(iii) *add the characteristic and the mantissa 1 + 0.30 = 1.30.*

Therefore, log 20 = 1.30.
Similarly, log 200 = 2.30;
and log 2000 = 3.30;
and log 55 = 1.74.

Example No. 4. *To find the log of the number 4,875, first correct to two significant figures.*

4,875 = 4,900.

Then, from the tables the mantissa is 0.68 and by inspection the characteristic is 3.

Therefore, log 4,875 = 3.68.
Similarly, log 281.65 = 2.45.

EXERCISES

1. From tables, find the logarithm of each of the following numbers:-

10^5; 45; 4.5; 9.02; 8,200; 761; 487,500.

2. What is the logarithm of:-

$\frac{900}{10}$, $\frac{420}{20}$, $\frac{1,250}{30}$, $\frac{1.5}{0.02}$, $\frac{0.006}{0.00025}$, $\frac{2,400}{0.05}$?

3. What is the value of 10 times the logarithm of:-

10^3; 1,000,000; 5,000; 860,000; 5.26; $\frac{850}{75}$?

2. POWER RATIOS.

2.1 Telecom apparatus usually produces either an effective increase or decrease in the level of the signal passing through it. Items such as amplifiers produce an increase (or gain) of signal level - the output power is greater than the input power. Items such as attenuators, pads, filters, equalisers and telephone lines produce a decrease (or loss) of level - the input power is greater than the output power.

It is customary to express the ratio between the input and the output powers in terms of a unit called the bel, which is equal to the logarithm of the power ratio.

A power ratio of 10 (or 10^1) is 1 bel;

100 (or 10^2) is 2 bels;

1,000 (or 10^3) is 3 bels, etc.

2.2 The decibel (abbreviated to dB) is the smaller practical unit and is $\frac{1}{10}$ of a bel.

1dB = 0.1 bel or 1 bel = 10dB.

The gain (or loss) in dB between the input power and the output power is equal to 10 times the log of the power ratio. Expressed as a formula:-

$$NdB = 10 \log \frac{P1}{P2}$$

where *NdB is the number of decibels gain or loss*

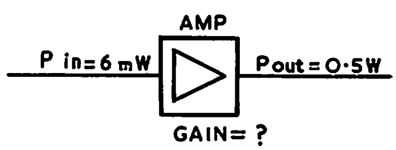
P1 and P2 are the two power levels expressed in the same unit.

For ease of calculation, the larger power is used as the numerator of the power ratio. Thus in the calculation of gain, P1 is the output power (with P2 the input); and in the calculation of loss, P1 is the input power (with P2 the output).

2.3 Example No. 5. What is the gain of an amplifier which delivers an output power of 0.5W when an input power of 6mW is applied?

Solution: As the amplifier has a gain, the output power (500mW) is the larger and becomes P1.

$$NdB = 10 \log \frac{P1}{P2}$$
$$= 10 \log \frac{500}{6}$$
$$= 10 \log 83$$
$$= 10 \times 1.92 = 19.2dB$$

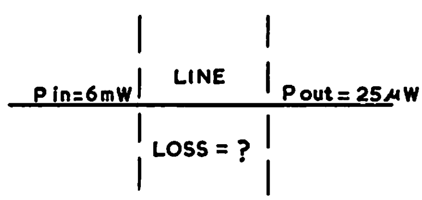


Answer : 19.2dB

2.4 Example No. 6. What is the attenuation in dB of a telephone line when the input power is 6mW and the power received at the distant end is 25uW?

Solution. As the line has a loss, the input power (6,000uW) is larger and becomes P1.

$$NdB = 10 \log \frac{P1}{P2}$$
$$= 10 \log \frac{6,000}{25}$$
$$= 10 \log 240$$
$$= 10 \times 2.38 = 23.8dB$$



Answer : 23.8dB

Note: Line transmission maintenance practices using quality control charts require the level measurements to be accurate to one decimal place.

EXERCISES

1. A transmission line has a signal of 2mW applied and the received signal at the distant end is 0.2mW. Find the loss over the line.
2. The output of an amplifier is 0.24 watts when 2mW is the value of power input. Calculate the gain of the amplifier.
3. When the input power to an amplifier is 0.1mW, the output is 10mW. Find the dB gain.
4. A power of 2.5mW is applied to a transmission line. Find the dB loss over the line if 0.5mW is available at the distant end.
5. An amplifier has a signal of 1mW applied and the output power is 25mW. What is the gain of the amplifier in dB?
6. A filter has a signal of 18mW applied to it and, at the output, the reading is 1.5mW. Calculate the loss of the filter.
7. Calculate the gain of an amplifier if the input signal is 2.25mW when the output signal is 45mW.
8. Calculate the attenuation in dB over a line which delivers an output signal of 0.01mW when 10mW is applied.
9. When the input power to a transmission line is 2.5mW and the output is 0.05mW, what is the loss over the line in dB?
10. What is the gain of an amplifier in dB when 2mW is applied to the input and the output power is 40mW?
11. The dB can be used to express a change of power (increase or decrease) at any point in a circuit. What is the change of power expressed in dB when the power applied to a telephone line -

    (i) is increased from 2mW to 3mW; (ii) is reduced from 2mW to 1mW?

    (Note that, although the power is altered by 1mW in each case, the power ratios and, therefore, the dB equivalents are different).
12. A public address amplifier delivers power to a system of four loudspeakers. It is required to increase the power delivered to each speaker to four times its former value. By how many dB must the gain of the amplifier be increased?
13. The instantaneous values of power in the output of an amplifier range between 0.1mW and 100mW. What is the dB equivalent for this power ratio?
14. Without reference to log tables, complete the following power ratios for each dB equivalent:-

Power Ratio	dB equivalent	Power Ratio	dB equivalent	Power Ratio	dB equivalent
	1		11		21
	2		12		22
2 : 1	3		13		23
	4		14		24
	5		15		25
	6		16		26
	7		17		27
	8		18		28
	9		19		29
10 : 1	10		20		30

<u>Remember</u> : A power ratio of 2:1 represents a 3dB change.

A power ratio of 10:1 represents a 10dB change.

3. USE OF ANTILOGARITHMS.

3.1 Antilogarithm tables list the numbers corresponding to a given logarithm and are to solve problems where the gain or loss of a circuit is given, and the input or output power is to be found. A simplified set of "two-figure" antilogarithm tables is shown at the end of this book.

When reading antilog tables, the mantissa is used to determine the figures in the required number; and the value of characteristic places the decimal point in these figures.

When the characteristic is 0, the number lies between 1 and 10.

When the characteristic is 1, the number lies between 10 and 100.

When the characteristic is 2, the number lies between 100 and 1,000, etc.

The number of figures to the left of the decimal point in the number is equal to the characteristic plus one.

Example No. 7. *To find a number which has a log of 0.57 (that is, the antilog of 0.57) look down the "Antilog" column until you reach 0.57. Read across to the right to find 37 in the "Number" column. The characteristic 0 indicates that the number is between 1 and 10.*

Therefore, antilog 0.57 = 3.7.

Similarly, antilog 0.3 = 2.0.

antilog 0.00 = 1.0.

Example No. 8. *To find the antilog of 1.57, again look down the "Antilog" column until you reach 0.57. Read across and to the right to find 37 in the "Number" column. The characteristic 1 in the value 1.57 indicates that the number lies between 10 and 100.*

Therefore, antilog 1.57 = 37.0.

Similarly, antilog 2.57 = 370;

antilog 3.57 = 3,700; and so on

antilog 1.00 = 10.

antilog 2.00 = 100.

antilog 3.00 = 1,000; and so on

3.2 Revision Exercises. From tables, find the antilog of the following:-

1. 0.25; 0.48; 0.89; 0.00; 0.33; 0.05; 0.2.

2. 2.25; 1.08; 2.64; 6.00; 3.33; 3.56; 2.99.

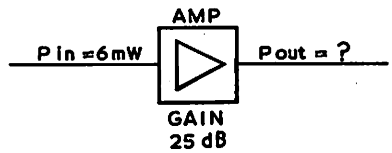
3. $\frac{75}{10}$; $\frac{25}{20}$; $\frac{6}{10}$; $\frac{10}{20}$; $\frac{27}{10}$; $\frac{20}{20}$; $\frac{3}{10}$;

3.3 When the power ratio of a circuit is known, the output power can be calculated for a given input, or the input power can be found for a known output.

Example No. 9. A power of 6mW is applied to an amplifier which has a gain of 25dB. Calculate the output power in watts.

Solution.

Formula .... $NdB = 10 \log \frac{P1}{P2}$



The output being the larger power becomes P1.

Substitute values ...... $25 = 10 \log \frac{P1}{6}$

Divide both sides by 10 ... $\frac{25}{10} = \frac{10}{10} \log \frac{P1}{6}$

$$2.5 = \log \frac{P1}{6}$$

Take antilog of both sides............

antilog $2.5 =$ *antilog* $(\log \frac{P1}{6})$

$$\begin{bmatrix} \text{antilog } 0.5 = 3.2 \\ \text{antilog } 2.5 = 320 \end{bmatrix}$$

Therefore .... $320 = \frac{P1}{6}$

$$P = 320 \times 6 = 1{,}920mW$$

Answer : 1.9 watts.

3.4 Further examples are given in paras. 3.6 and 3.7 of the Course Paper, "Transmission Units".

EXERCISES

1. An amplifier has a gain of 6dB. If the input power is 1mW, what is the value of output power?

2. The gain of an amplifier is 15dB. What is the output power when the input is 2mW?

3. What power is available at the distant end of a line when the input signal is 30mW, and the attenuation of the line is 10dB?

4. The gain of an amplifier is 30dB. If the signal applied is 1mW, calculate the output power.

5. If the signal at the distant end of a transmission line which has a loss of 17dB, is 2mW, calculate the value of the input power.

6. A signal of 1mW is applied to an amplifier, which has a gain of 29dB. Calculate the value of the output power.

7. The loss over a circuit is 23dB. If the input signal is 39.9mW, find the value of the output power in microwatts.

8. A line has a loss of 17dB. What is the input power when the output power is 0.1mW?

9. Calculate the power in watts which is 33dB greater than 1mW.

10. The power applied to an amplifier is 0.1mW. To what value should this power be altered to increase the output power by 10dB?

4. OVERALL GAIN OR LOSS.

4.1 When a circuit consists of a number of items of equipment (such as a carrier channel or a long trunk line with repeater stations) the signal is subjected to both power gains and losses.

The overall gain or loss of the circuit is expressed in dB and is the difference between the sum of the gains and the sum of the losses.

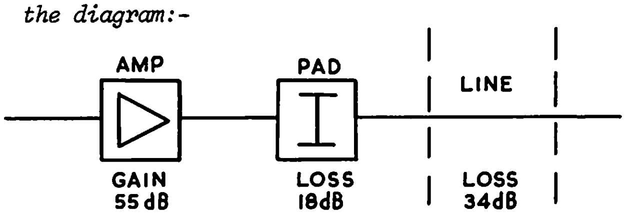
The circuit has an overall gain when the total gain exceeds the total loss.

The circuit has an overall loss when the total loss exceeds the total gain.

A circuit has zero loss when the sum of the gains equals the sum of the losses.

4.2 Example No. 10. A circuit consists of an amplifier with a gain of 55dB connected to a pad which has a loss of 18dB and a line presenting a further loss of 34dB. Draw a simple diagram to illustrate this circuit and calculate the overall loss or gain.

Solution : Draw the diagram:-



Total gain = 55dB.

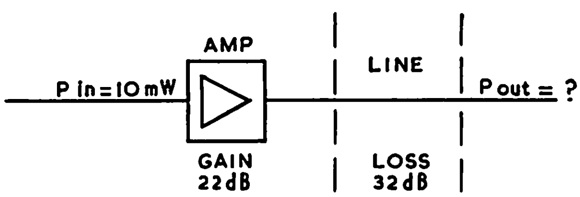
Total loss = 18 + 34 = 52dB.

Total gain exceeds total loss; therefore -

Overall gain = 55 - 52 = 3dB.

Answer : 3dB overall gain.

4.3 Example No. 11. In the diagram, the power input to the amplifier is 10mW. Find the power output at the distant end of the line. (As a first step, calculate overall loss or gain).



Solution:

$$NdB = 10 \log \frac{P1}{P2}$$

$$10 = 10 \log \frac{10}{P2}$$

$$1 = \log \frac{10}{P2}$$

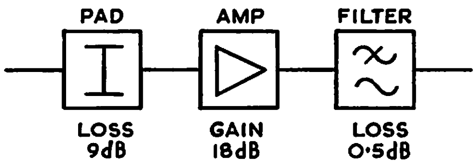
$$10 = \frac{10}{P2}$$

$$P2 = \frac{10}{10} = 1mW.$$

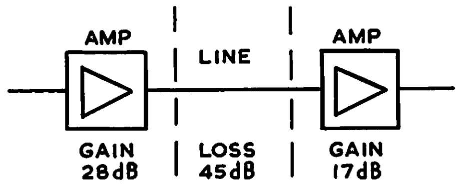
Answer : 1mW.

EXERCISES

1-2. Calculate the overall loss or gain in each of the following circuits.

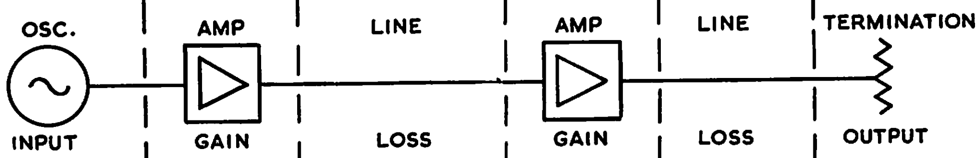


(1)



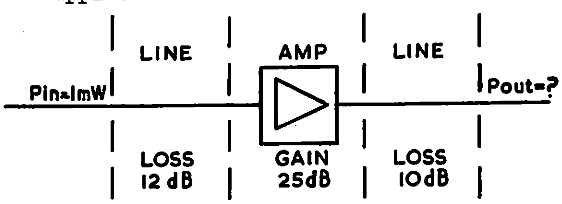
(2)

3-7. Fill in the missing spaces in the following chart.

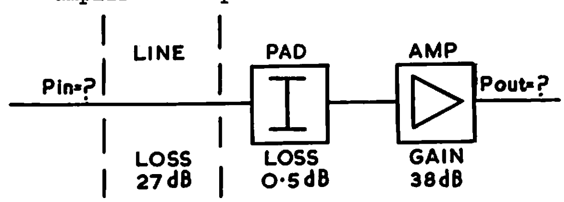


	INPUT	GAIN	LOSS	GAIN	LOSS	OUTPUT
3.	10mW	15dB	30dB	20dB	10dB	
4.	0.5mW	22dB	17dB	18dB	16dB	
5.	2mW	10dB	15dB		7dB	3.2mW
6.	1mW		19dB	17dB	3dB	160mW
7.	2mW	16dB		16dB	17dB	1mW

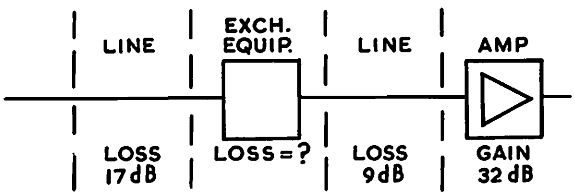
8. Calculate the output power from the circuit when 1mW is applied.



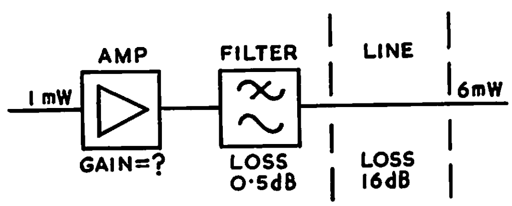
9. Calculate the signal power input to the line to provide for 1mW at the amplifier output.



10. Calculate the loss through the exchange equipment, when the circuit has zero loss.



11. Calculate the gain of the amplifier.



5. REFERENCE POWER LEVEL - THE dBm.

5.1 Telecom measurements and calculations are simplified when power levels throughout a circuit are expressed with respect to a reference power level. The standard reference level is generally 1mW and the power level at any point in a circuit can then be expressed in decibels with respect to 1mW. To express a power level in this manner the abbreviation dBm is used, and it signifies "dB with respect to a reference level of 1mW".

Power levels greater than 1mW must have a positive (+) sign before the dBm value.

Power levels less than 1mW must have a negative (-) sign before the dBm value.

Any value given in dBm is therefore an expression of a power level; and power levels expressed in dBm are independent of the impedance of the circuit, and of the frequency.

Examples:
(i) 10mW is "10dB greater than 1mW" or +10dBm.
(ii) 1 watt (1,000mW) is "30dB greater than 1mW" or +30dBm.
(iii) 0.1mW is "10dB less than 1mW" or -10dBm.
(iv) 1μW (0.001mW) is "30dB less than 1mW" or -30dBm.
(v) 1mW is expressed as 0dBm (zero dBm).

5.2 To convert power in mW to power level in dBm, use the dB formula -

$$NdB = 10 \log \frac{P1}{P2}$$

When the power is greater than 1mW, make P1 the power in mW, and P2 the reference level, 1mW. The answer must be expressed as +dBm.

When the power is less than 1mW, make P1 the reference level in mW, and P2 the known power. The answer must be expressed as -dBm.

5.3 Example No. 12. (i) *To express 8mW as dBm -*

$$NdB = 10 \log \frac{P1}{P2}$$

$$= 10 \log \frac{8}{1}$$

$$= 10 \times 0.90 = 9.0$$

Answer : +9dBm.

(ii) *to express 0.2mW as dBm -*

$$NdB = 10 \log \frac{P1}{P2}$$

$$= 10 \log \frac{1}{0.2} = 10 \log 5.$$

$$= 10 \times 0.70 = 7.0$$

Answer : -7dBm.

5.4 To convert power level in dBm to power in mW, again use the dB formula.

For levels below 1mW (-dBm), P1 is 1mW and P2 is the unknown value.

For levels above 1mW (+dBm), P1 is the unknown value and P2 is 1mW.

5.5 Example No. 13. (i) *Express +13dBm in mW.*

$$NdB = 10 \log \frac{P1}{P2}$$

$$13 = 10 \log \frac{P1}{1}$$

$$1.3 = \log P1$$

$$\text{antilog } 1.3 = P1$$

$$P1 = 20$$

Answer : 20mW

(ii) *Convert -24dBm to μW.*

$$NdB = 10 \log \frac{P1}{P2}$$

$$24 = 10 \log \frac{1}{P2}$$

$$2.4 = \log \frac{1}{P2}$$

$$\text{antilog } 2.4 = \frac{1}{P2} = 250$$

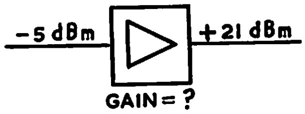
$$\text{Therefore } P2 = \frac{1}{250}$$

Answer : 4μW

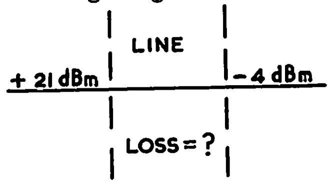
5.6 Further examples are given in paras. 5.5 to 5.8 of the Course Paper, "Transmission Units".

EXERCISES

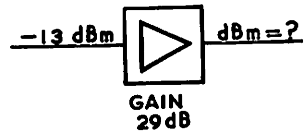
1. Find the missing values in the following diagrams.



(i)



(ii)



(iii)

2. Calculate the output level in dBm when a power of 1mW is applied to an amplifier with a gain of 25dB.

3. What power level in dBm must be supplied to a 3dB pad to give 2mW in the output?

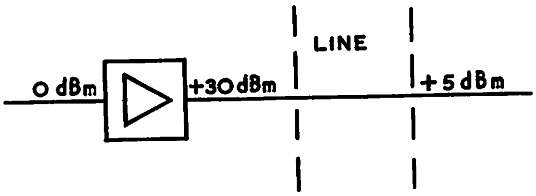
4. What input level in dBm must be applied to an amplifier with a gain of 29dB to give an output power of 2.5 watts?

5. Calculate the gain of an amplifier which delivers an output power of 0.5 watts when the input signal level is -4dBm.

6. When the gain of an amplifier is increased by 3dB, the output level is measured at +30dBm. What was the power output in mW of the amplifier before increasing the gain?

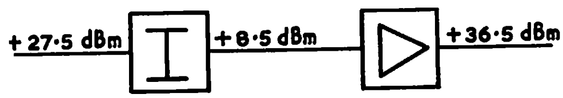
7. Calculate:-

(i) the gain of the amplifier;
(ii) the loss over the line;
(iii) the overall loss or gain.



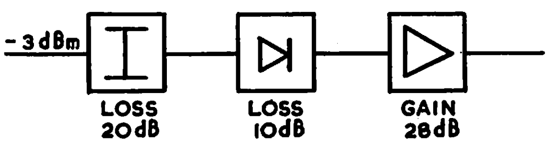
8. Calculate:-

(i) the loss in the pad;
(ii) the gain of the amplifier;
(iii) the overall loss or gain.



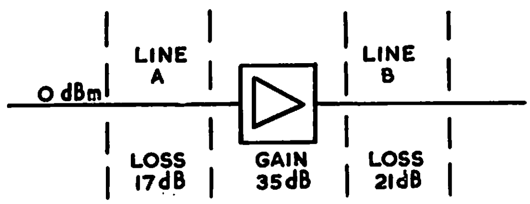
9. Calculate the power in mW at:-

(i) the input to the pad;
(ii) the input to the modulator;
(iii) the input to the amplifier;
(iv) the output of the amplifier;



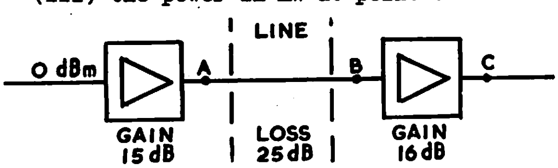
10. Calculate:-

(i) the overall loss or gain;
(ii) power in mW at input to amplifier;
(iii) power level in dBm at output of amplifier.



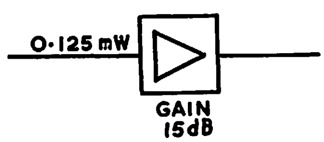
11. Calculate:-

(i) the power level in dBm at point A;
(ii) the power level in dBm at point B;
(iii) the power in mW at point C.



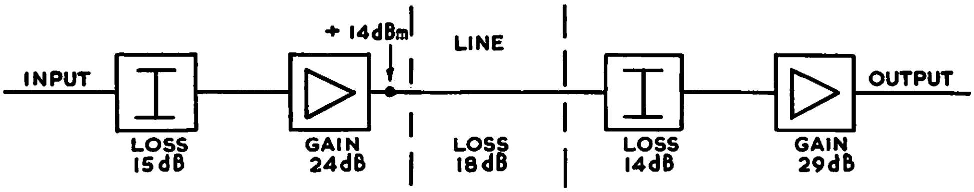
12. Calculate:-

(i) the input level in dBm;
(ii) the output power in mW;
(iii) the output level in dBm.

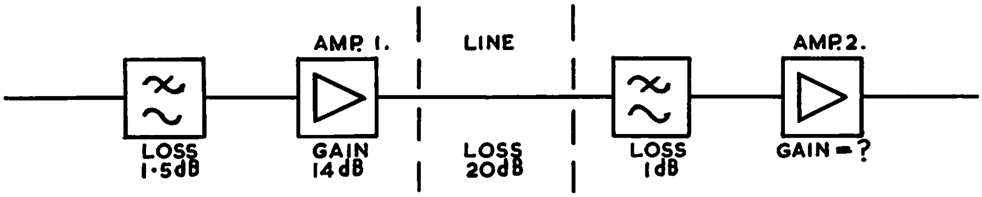


EXERCISES

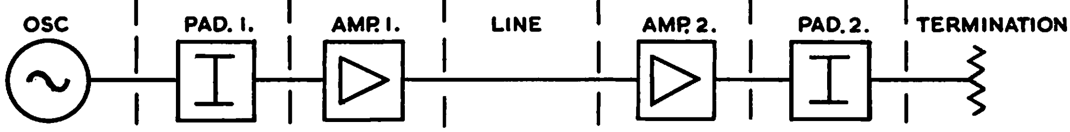
13. Calculate input and output levels in dBm.



14. Calculate the gain of amplifier 2, if the overall loss of the circuit is 4dB.

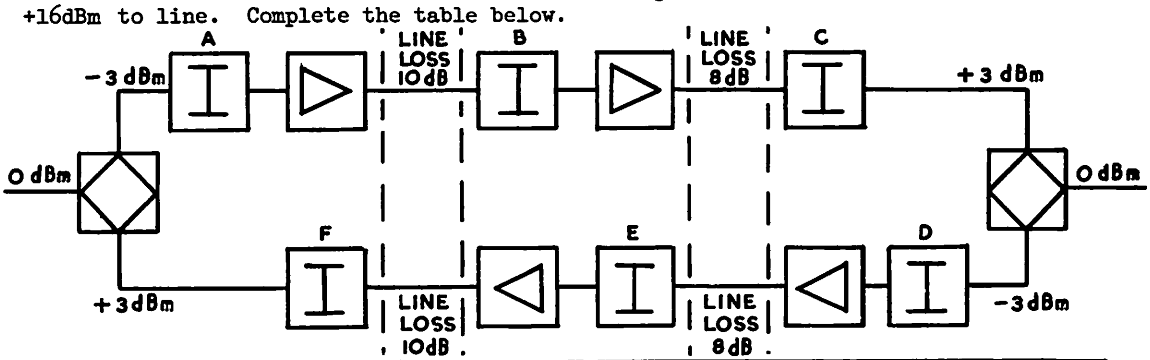


15-20 Complete the following table by inserting your answer in the blank space provided.



	Oscillator Output	Loss of Pad 1	Gain of Amp. 1	Loss over Line	Gain of Amp. 2	Loss of Pad 2	Power in Termination
15.	0dBm	10dB	16dB	16dB		10dB	0dBm
16.	+6dBm		18dB	15dB	20dB	12dB	0dBm
17.		11dB	40dB	12dB	30dB	20dB	+3dBm
18.	-6dBm	5dB	10dB		27dB	11dB	0dBm
19.	-6dBm	11dB	40dB	18dB	10dB	15dB	
20.	-3dBm	10dB		12dB	15dB	8dB	+2dBm

21. In the following diagram, each amplifier has a gain of 26dB, and the circuit has a "zero" overall loss. Calculate the loss in each pad to allow each amplifier to send +16dBm to line. Complete the table below.



Pad	A	B	C	D	E	F
Loss in dB.						

6. POWER FORMULAS AND THE dB.

6.1 In some dB problems, we first have to calculate the input and output powers by applying the power formulas -

$$P = EI \qquad\qquad P = I^2R \qquad\qquad P = \frac{E^2}{R}$$

These formulas are used to calculate the power in non-reactive circuits which have zero phase angle. In dB problems, it is generally assumed that the input and output impedances of circuits are non-reactive.

6.2 Example No.14. The input and output impedances of an amplifier are 600 ohms and 16 ohms respectively. If an input signal at +8dBm is applied, what is the current in a 16 ohm load when the gain of the amplifier is 25dB?

Solution. *First calculate the power dissipated in the load; then apply the appropriate power formula to find the current in the load.*

Output power level = + 8 + 25 = +33dBm.

Therefore, output power is 33dB greater than 1mW.

Using the dB formula, output power = 2 watts.

Then, by transposition current in load $= \sqrt{\frac{P}{R}} = \sqrt{\frac{2}{16}} = 0.35A$

Answer : 0.35 ampere.

EXERCISES

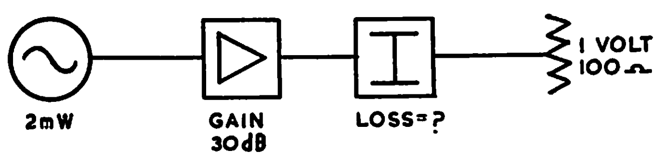
1. A signal voltage of 0.3V is fed to an amplifier with an input impedance of 150 ohms and an output impedance of 600 ohms. If the output voltage is 12V, calculate the gain of the amplifier in dB.

2. A pad with an input impedance of 50 ohms and an output impedance of 600 ohms is connected to a 600 ohm termination. If the signal voltage applied to the input is 1 volt and the current in the termination is 1mA, calculate the loss of the pad in dB.

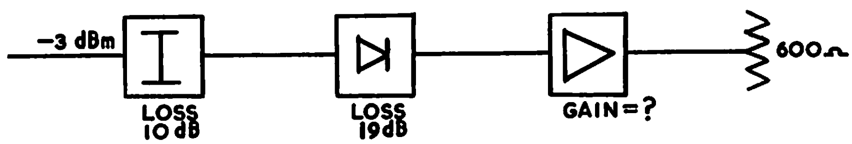
3. In the following circuit, calculate:-

   (i) the power level in dBm at the output of the amplifier;

   (ii) the loss in the attenuator.



4. (i) Calculate the gain of the amplifier when the voltage at the termination is 1.5V.



   (ii) Calculate the increase required in the gain of the amplifier to obtain 2V at the termination.

5. With reference to Example No. 14 in para. 6.2, calculate the voltage across the load when the gain of the amplifier is reduced by 9dB.

7. VOLTAGE AND CURRENT RATIOS.

7.1 Although the dB is fundamentally a unit of power ratio, gain or loss in dB can be calculated from the voltage or current ratio, when the impedances are equal. Use the formulas -

$$NdB = 20 \log \frac{E1}{E2} \qquad NdB = 20 \log \frac{I1}{I2}$$

(These formulas are derived from the basic power ratio formula - refer Course Paper, "Transmission Units", para. 4.1).

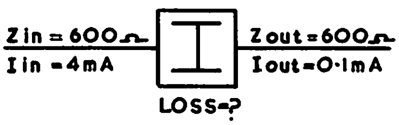
Remember. The current or voltage ratios are used only for matched impedances, that is, when the input and output impedances are equal. When these impedances are not equal, the basic power ratio formula must be used.

7.2 Example No. 15. The input and output impedances of an attenuator are 600 ohms. If an oscillator supplies an input current of 4mA and the current through the 600 ohm load is 0.1mA, what is the loss of the attenuator?

Solution. As the input and output impedances are equal, we can use the current ratio formula to calculate the dB loss. For ease of calculation, put the larger (input) current as the numerator.

$$NdB = 20 \log \frac{I1}{I2}$$

$$= 20 \log \frac{4}{0.1}$$



$$= 20 \log 40 = 20 \times 1.60 = 32dB.$$

Answer: 32dB.

7.3 Example No. 16. The signal voltage measured across the input terminals of an amplifier having equal input and output impedances is 0.07 volt. To what value should the input voltage be varied to increase the output power by 12dB?

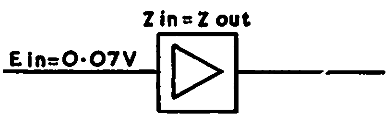
Solution. To increase the output power by 12dB, the input power is increased by the same amount. The problem requires us to find a voltage which is 12dB greater than 0.07 volt across the same input impedance. We can use the voltage ratio formula -

$$NdB = 20 \log \frac{E1}{E2}$$

As the unknown voltage is the larger, make this E1.

$$12 = 20 \log \frac{E1}{0.07}$$

$$0.6 = \log \frac{E1}{0.07}$$



Therefore, $\frac{E1}{0.07} = \text{antilog } 0.6 = 4.0$

and $E1 = 0.07 \times 4.0 = 0.28$ *volt*

Answer: 0.28 volt.

7.4 Further examples are given in paras. 4.3 and 4.4 of the Course Paper, "Transmission Units".

EXERCISES

1. What is the gain of an amplifier with equal input and output impedances when the voltage measured across the input terminals is 0.07 volts and the output voltage is 560mV?

2. In Example No. 15, para. 7.2, calculate the new value of current in the load when the input current is increased to 8mA.

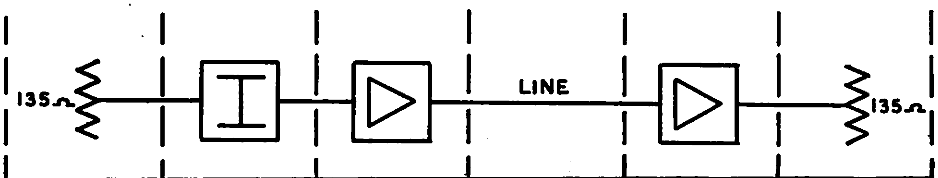
3. A signal voltage of 0.5 volts is applied to an amplifier which has equal input and output impedances. If the output voltage is 7.5V, calculate the gain of the amplifier.

4. An amplifier with equal input and output impedances is connected to a 2.5V, 1000c/s oscillator. Calculate the voltage at the output when the gain of the amplifier is - (i) 6dB; (ii) 12dB; (iii) 15dB; (iv) 23dB;

5. A filter with equal input and output impedances has a current of 0.07mA in the input circuit and a current of 50μA in the output circuit. What is the loss of the filter in dB?

6. The input and output impedances of a 6dB pad are 600 ohms. If the input current is 0.1mA what is the value of the output current in μA?

7-11 Fill in the missing spaces in the following chart:-



	Input	Loss of Pad	Gain of Amp. 1	Loss of Line	Gain of Amp. 2	Output
7	1mV	22dB	13dB	15dB	33dB	....mV
8	...mA	18dB	38dB	16dB	14dB	8mA
9	0.05A	8dB	29dB	24dB	...dB	625μA
10	14mV	11dB	28dB	...dB	26dB	0.35V
11	1mA	...dB	18dB	20dB	27dB	10mA

12. Without reference to log tables, complete the following voltage and current ratios for each dB equivalent.

Voltage or Current Ratio	dB equivalent	Voltage or Current Ratio	dB equivalent
	6		8
	12		14
	18		20
	24		26
	30		32
	36		38

Remember. With equal impedances:-

A voltage or current ratio of 2:1 represents a 6dB change.

A voltage or current ratio of 10:1 represents a 20dB change.

4. THE MAGNETIC CIRCUIT

Reference: ETP 0300, "Magnetism and Electromagnetism" Section 4

1. COMPARISON WITH THE ELECTRIC CIRCUIT

1.1 A MAGNETIC CIRCUIT is the path along which magnetic lines of force can be established.

An ELECTRIC CIRCUIT is the path through which an electric current passes.

In telecom apparatus, magnetism and electricity are so closely related that the same fundamental laws apply to both types of circuit. It is merely necessary, therefore, to apply the laws of the electric circuit as restated in the terms and units of the magnetic circuit.

1.2 When a coil of wire carries an electric current, it develops a force (magnetomotive force) to establish magnetic lines of force (magnetic flux) against the opposition (reluctance) offered in the magnetic circuit. In the electric circuit, an electromotive force sets up an electric current against the resistance of the circuit.

(a) MAGNETIC CIRCUIT. (b) ELECTRIC CIRCUIT.

Magnetic Term and Symbol	*Basic Unit and Symbol*	*Electric Term and Symbol*	*Basic Unit and Symbol*
Magnetomotive force (m.m.f.) (F)	*Ampere* (A)	*Electromotive force (e.m.f.)* (E)	*Volt* (V)
Reluctance (R)	*Reciprocal Henry* (H^{-1})	*Resistance* (R)	*Ohm* (Ω)
Magnetic flux (ϕ)	*Weber* (Wb)	*Electric current* (I)	*Ampere* (A)
Flux = $\frac{\text{Magnetomotive force}}{\text{Reluctance}}$ or $\phi = \frac{F}{R}$		*Current* = $\frac{\text{Electromotive force}}{\text{Resistance}}$ or $I = \frac{E}{R}$	

1.3 A magnetic circuit may consist of non-magnetic material (for example, wood, brass, copper) wound with a number of turns of insulated wire. When an electric current flows in the coil, a magnetomotive force is developed and magnetic lines of force are set up in the material. When the coil has an air core or is wound on a core of non-magnetic material, the reluctance is very high and the flux in the magnetic circuit is relatively weak.

When a core of magnetic material (for example, soft iron or steel) is used in the magnetic circuit, this appears to offer a much easier path or lower reluctance) for the lines of force than does the air, and the flux is increased. Materials with this property are said to have a high relative permeability compared to air or other non-magnetic materials.

When an air-gap is introduced in the magnetic circuit (for example, the air gap between armature and core in the case of a telephone relay), the reluctance of the air-gap is much greater than that of the magnetic material. This has an important bearing on the value of flux in the magnetic circuit and, therefore, on the operating performance of the relay.

2. MAGNETOMOTIVE FORCE

2.1 MAGNETOMOTIVE FORCE (m.m.f.) is the force which establishes and maintains the flux in the magnetic circuit. The m.m.f. may be caused by a permanent magnet or by an electric current flowing in a conductor or in a coil.

The magnetomotive force (F) of a coil is proportional to the current (I) and to the number of turns (N).	$F \propto IN$

2.2 UNIT AND FORMULA. The value of m.m.f. is expressed by the product of the current in amperes and the number of turns; for many years, the unit was the "ampere-turn." More recently, the word "turn" has been omitted from this unit, and the unit of m.m.f. is now the AMPERE (symbol A).

$F = IN$ *where* *F is m.m.f. in amperes (magnetic)*
I is current in amperes (electric)
N is number of turns.

2.3 EXAMPLE No. 1. What is the m.m.f. developed by 6,000 turns of wire carrying an electric current of 40mA?

SOLUTION $F = IN$
$= 0.04 \times 6{,}000 = 240$ *amperes*
ANSWER: *240A*

Note that the m.m.f. produced by an electric current of 40mA flowing through 6,000 turns, is equivalent to that produced by an electric current of 240A flowing through 1 turn. The numerical value of m.m.f. in amperes, therefore, indicates that value of electric current which would be required to flow through 1 turn, in order to produce the equivalent m.m.f.

EXERCISES

1. Assuming other factors constant, how is the m.m.f. in the magnetic circuit of a telephone relay affected when -

   (i) the number of turns is doubled;
   (ii) the current in the relay winding is halved;
   (iii) the voltage across the winding is halved;
   (iv) the resistance of the winding is doubled;
   (v) the number of turns and the resistance of the winding are both doubled?

2. Referring to Example No. 1, what is the m.m.f. in amperes, when -

   (i) the number of turns is reduced to 4,000;
   (ii) the current is increased to 60mA?

3. What is the m.m.f. in amperes, in the magnetic circuit of a telephone relay with 4,000 turns carrying 60mA?

4. What current must flow through a coil of 5,000 turns to produce an m.m.f. of 250A?

5. A current of 40mA flows through a relay winding. How many turns are required to produce a magnetomotive force of 200A?

3. RELUCTANCE

3.1 RELUCTANCE is the opposition a material offers to the passage of magnetic flux.

The reluctance (R) of any part of a magnetic circuit is directly proportional to its length (l), and inversely proportional to its cross-sectional area (a) and the relative permeability (μ_r) of the material. Note the similarity between this rule and that for the resistance of a conductor in an electric circuit:- $R \propto \frac{\rho l}{a}$	$R \propto \frac{l}{\mu_r a}$

3.2 UNIT AND FORMULA. The unit of reluctance is the RECIPROCAL HENRY (symbol H^{-1}). In some text books, this unit may be called Ampere/Weber (symbol A/Wb). This is the reluctance of a magnetic circuit in which an m.m.f. of 1 ampere produces 10^8 lines of force.

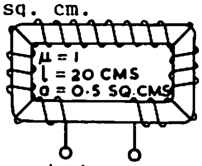
$R = k(\frac{l}{\mu_r a})$ *where*

R is reluctance in reciprocal henrys,
l is length of magnetic circuit in metres,
a is cross-sectional area in square metres,
μ_r is relative permeability of the magnetic circuit,
k is a constant value $= \frac{1}{4\pi \times 10^{-7}}$

NOTES. 1. To simplify the solution of problems, the value of k may be approximated to 8×10^5. This is sufficiently accurate as the error is less than 1%.

2. In magnetic circuit problems, the relative permeabiliuty (see Section 7) of non-magnetic substances, including air, is taken as 1. The relative permeability of magnetic substances is very much greater than 1; for certain nickel-iron alloys, it may be as high as 100,000.

3.3 EXAMPLE No. 2. What is the reluctance in reciprocal henrys of a uniform magnetic circuit consisting of non-magnetic material, 20cm. long and 0.5 sq. cm. cross-sectional area?



SOLUTION: $R = \frac{8 \times 10^5 \times l}{\mu_r a} = \; = \frac{8 \times 10^5 \times 20 \times 10^{-2}}{1 \times 0.5 \times 10^{-4}}$

ANSWER: $= 320 \times 10^7$ reciprocal henrys or $3.2 \times 10^9\ H^{-1}$ (or A/Wb)

1. Assuming other factors constant, how is the reluctance of an air gap of circular cross-sectional area in a magnetic circuit affected, when -

   (i) the length is halved;
   (ii) the cross-sectional area is doubled;
   (iii) the diameter is doubled?

2. Referring to Example No. 2, what is the reluctance in reciprocal henrys of the magnetic circuit when -

   (i) the length is increased to 25 cm;
   (ii) the cross-sectional area is increased to 2 sq. cm?

3. In Example No. 2, what is the reluctance if the non-magnetic material is replaced by a magnetic material with a relative permeability of 800?

4. MAGNETIC FLUX

4.1 MAGNETIC FLUX is the total number of lines of force in a magnetic circuit.

In a magnetic circuit, the flux (ϕ - Greek letter, Phi) is directly proportional to the magnetomotive force (F) and inversely proportional to the reluctance (R). (Note the similarity between this rule and Ohms Law, $I \propto \frac{E}{R}$.)	$\phi \propto \frac{F}{R}$

Combining the rules for m.m.f., reluctance and flux -

$$\phi \propto \frac{IN}{R} \quad ; \quad \phi \propto \frac{IN\mu_r a}{l}$$

4.2 UNITS AND FORMULAS. The unit of magnetic flux is the WEBER (symbol Wb). In practice, the sub-multiple units, milliweber (mWb) and microweber (μWb) are often used. One weber is equivalent to 100,000,000 = 10^8 lines of force.

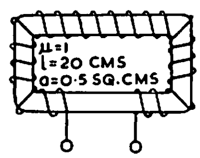
$$\phi = \frac{F}{R}$$

where ϕ *is magnetic flux in Webers;*
F *is magnetomotive force in amperes,*
R *is reluctance in reciprocal henrys.*

When either F or R is unknown, transform the formula, thus:-

$$F = \phi R \qquad\qquad R = \frac{F}{\phi}$$

4.3 EXAMPLE No. 3. Calculate the flux in a uniform magnetic circuit consisting of non-magnetic material (20 cm. long and 0.5 sq. cm. cross-sectional area) in which a magnetomotive force of 240 amperes is developed and which has a reluctance of 3.2×10^9 reciprocal henries. (Note: This example combines the values given in Examples Nos. 1 and 2.)



$I = 40mA$
$N = 6000$

SOLUTION: $\phi = \frac{F}{R}$

$$= \frac{240 \times 10^6}{3.2 \times 10^9} = \frac{75}{10^3} = 0.075\mu Wb \text{ Answer.}$$

EXERCISES

1. Assuming other factors constant, how is the flux in a uniform magnetic circuit of circular cross-sectional area, affected when -

    (i) the number of turns is doubled;
    (ii) the current in the winding is halved;
    (iii) the length of the flux path is halved;
    (iv) the cross-sectional area is doubled;
    (v) the diameter is doubled;
    (vi) the relative permeability is increased?

2. Referring to Exmaples No. 3, calculate the value of flux in the magnetic circuit, when:-

    (i) the m.m.f. is increased to 320A;
    (ii) the reluctance is reduced to $2.4 \times 10^9\ H^{-1}$.

3. In Example No. 3, what is the new value of flux if the non-magnetic material is replaced with a magnetic material with a relative permeability of 800?

5. MAGNETISING FORCE

5.1 MAGNETISING FORCE (field strength or field intensity) is the magnetomotive force per metre length of a uniform magnetic circuit.

In comparison to an electric circuit, m.m.f. can be likened to the applied e.m.f. whilst the magnetising force is compared with a P.D. across one part of the circuit.

In a magnetic circuit, the magnetising force (H) is directly proportional to the magnetomotive force (F) and inversely proportional to the length (l).	$H \propto \frac{F}{l}$

Combining the rules for m.m.f. and magnetising force:-

$$H \propto \frac{IN}{l}$$

5.2 UNIT AND FORMULA. The unit of magnetising force is the AMPERE PER METRE. (A/m).

$$H = \frac{F}{l}$$ *where* *H is magnetising force in amperes per metre.*
F is magnetomotive force in amperes.
l is length of magnetic circuit in metres.

When the magnetic circuit is uniform (composed entirely of the same material of constant cross-sectional area), the value of H is the same all round the circuit, and the total values of F and l can be substituted in the formula.

When the magnetic circuit consists of two or more parts (for example, an air and an iron path), the values of H along each of the parts may be different, and the values of F and l are taken from the part of the circuit under consideration.

5.3 EXAMPLE No. 4. Find the magnetising force in the uniform magnetic circuit of Example No. 3 where the magnetomotive force is 240 amperes and the length is 20 cm.

SOLUTION: $H = \frac{F}{l} = \frac{240}{0.2} = 1{,}200$ *amperes per metre.*

ANSWER: 1,200A/m.

EXERCISES

1. Assuming other factors constant, how is the magnetising force in a magnetic circuit affected when-

   (i) the number of turns is doubled;
   (ii) the current in the winding is doubled;
   (iii) the voltage across the winding is halved;
   (iv) the length of the flux path is increased?

2. Referring to Exmaple No. 4, what is the value of magnetising force, when-

   (i) the m.m.f. is reduced to 180A;
   (ii) the length of the magnetic circuit is increased to 30cm?

3. What value of magnetomotive force is required to produce a magnetising force of 2,000 amperes per metre in a uniform magnetic circuit which has a length of 15cm?

4. A metal ring with an average circumference of 10cm. is wound with 10,000 turns of wire carrying an electric current of 20mA. Calculate the magnetising force in the magnetic circuit.

6. FLUX DENSITY

6.1 FLUX DENSITY (or degree of magnetisation) is a measure of lines of force per unit cross-sectional area of the magnetic circuit.

In a magnetic circuit, the flux density (B) is directly proportional to the magnetic flux (Ø) and inversely proportional to the cross-sectional area (a).	$B \propto \frac{\emptyset}{a}$

6.2 UNIT AND FORMULA. The unit of magnetic flux density is the TESLA (symbol T). One tesla is equal to one weber (10^8 lines of force) per square metre.

$$B = \frac{\emptyset}{a}$$

B is flux density
Ø is magnetic flux in webers.
a is cross-sectional area of magnetic circuit in square metres.

When the magnetic circuit consists of parts with different cross-sectional areas, note that the total flux is the same in each part (neglecting leakage), but the flux densities are different.

6.3 EXAMPLE No. 5. Calculate the flux density in the uniform magnetic circuit of Example No. 3 where the magnetic flux is 0.075 microweber and the cross-sectional area is 0.5 sq. cm.

SOLUTION:
$$B = \frac{\emptyset}{a}$$
$$\frac{0.075 \times 10^{-6}}{0.5 \times 10^{-4}} = 0.15 \times 10^{-2} \text{ tesla.}$$

ANSWER: 0.0015T

EXERCISES

1. Assuming other factors constant, how is the flux density in a magnetic circuit affected when -

   (i) the current in the winding is doubled;
   (ii) the voltage across the winding is halved;
   (iii) the reluctance is decreased;
   (iv) the length of the flux path is increased;
   (v) the cross-sectional area is increased;
   (vi) the permeability is increased?

2. What is the flux density in a magnetic circuit with a cross sectional area of 1.5 sq. cm. carrying a flux of 60 μWb?

3. What is the total flux in a sample of soft iron 1.5cm. x 2 cm. cross-section when the flux density is 2 teslas?

7. RELATIVE PERMEABILITY

7.1 When the non-magnetic material in a magnetic circuit is replaced by a magnetic material, the flux produced by a given m.m.f. is greatly increased. The ratio of the flux density (that is, the magnetic flux per square metre of cross-sectional area) produced in a material to the flux density produced in a non-magnetic material by the same magnetising force is called the relative permeability.

7.2 EXAMPLE No. 6. A sample of soft iron, when subjected to a magnetising force capable of creating in air a flux density of 0.0015 tesla, is found to have a flux density of 1.8 teslas. What is the relative permeability of the soft iron at this stage of the magnetisation?

SOLUTION: $\mu_r = \frac{B1}{B2}$ *where* μ_r *is relative permeability (no unit).* $B1$ *is flux density in the magnetic circuit.* $B2$ *is flux density in the non-magnetic circuit.*

$$= \frac{1.8}{0.0015} = 1,200$$

ANSWER: 1,200.

7.3 FORMULA. When the magnetising force and the related flux density in a magnetic circuit are known, the formula used to calculate the relative permeability of the magnetic material, is:-

$$\mu_r = k\left(\frac{B}{H}\right)$$ *where*

μ_r *is relative permeability (no unit).*

k *is a constant value* $= 8 \times 10^5$ *(approx).*
B *is flux density in teslas.*
H *is magnetising force in amperes per metre.*

7.4 EXAMPLE No. 7. When the magnetising force in a magnetic circuit is 1,200 amperes per metre, the flux density is 1.5 teslas. What is the relative permeability of the magnetic material for this value of magnetising force?

SOLUTION: $\mu_r = k\left(\frac{B}{H}\right) = \frac{8 \times 10^5 \times 1.5}{1,200} = 1,000$

ANSWER: 1,000

NOTE. *This value of relative permeability applies only for a magnetising force of 1,200 amperes per metre. In practice, the relative permeability of a magnetic material is not constant, but varies with different values of magnetising force and the state of magnetisation of the material.*

EXERCISES

1. What is the relative permeability of a soft iron ring in which a flux density of 2.4 teslas is produced by a magnetising force which is capable of producing a flux density of 0.0005 tesla in a copper ring?

2. Referring to Example No. 7, calculate the new values of relative permeability for the magnetic material, when:

   (i) a magnetising force of 600 amperes/metre produces a flux density of 0.9 tesla;

   (ii) a magnetising force of 1,800 amperes/metre produces a flux density of 1.8 teslas.

3. (i) The flux in 0.1 sq. metre cross-section of air is 20 μWb. Calculate:-
   (i) the flux density; (ii) the magnetising force.

   (ii) If a piece of iron ($\mu_r = 250$) is placed in the magnetic field referred to in (i), what is the flux density in the iron?

4. What is the strength of a magnetic field (in amperes per metre) when a bar of soft iron which is placed in it, has an induced flux density of 1.5 teslas. (Assume μ_r *of the soft iron is 750.)?*

5. Using the formulas for R, $\emptyset$, H and B given in paras, 3.2, 4.2, 5.2 and 6.2 respectively, derive a formula for relative permeability in terms of B and H.

8. THE ELECTROMAGNET

8.1 An electromagnet consists of a core of soft iron (or other magnetic material) with turns of insulated wire wound around it. Compared to a core of non-magnetic material, the magnetic core reduces the reluctance of the magnetic circuit, and increases the total flux and flux density to an extent governed by the relative permeability of the material used.

8.2 Example No. 7 A soft iron core 20 cm. long and 0.5 sq. cm. cross-sectional area, is wound with 6,000 turns of wire. If the relative permeability of the iron is 1,000 and a current of 40mA passes through the coil, calculate -

(i) the reluctance;
(ii) the total flux;
(iii) the flux density in the magnetic circuit.

(Note: This example uses the values given in the previous examples, except that a magnetic material of much higher relative permeability is used in the magnetic circuit)

Solution:

(i) $R = k(\frac{l}{\mu_r a}) = \frac{8 \times 10^5 \times 20 \times 10^4}{100 \times 10^3 \times 0.5} = \frac{8 \times 2 \times 10^6}{5} = 3.2 \times 10^6 H^{-1}$

(ii) $\emptyset = \frac{NI}{R} = \frac{6 \times 10^3 \times 40}{10^3 \times 3.2 \times 10^6} = \frac{240}{3.2 \times 10^4} = 75\mu Wb$

(iii) $B = \frac{\emptyset}{a} = \frac{75 \times 10^{-6}}{0.5 \times 10^{-4}} = \frac{75 \times 10^{-1}}{5} = 1.5T$

Answers: (i) 3.2×10^6 reciprocal Henry (ii) 75 μWb (iii) 1.5 Tesla

Compare these answers with those obtained in Examples Nos. 2, 3 and 5 for u = 1 -

($3.2 \times 10^9 H^{-1}$); (0.075uWb); (0.0015T)

Note that, when all other factors in a uniform magnetic circuit are kept constant -

(i) the reluctance is inversely proportional to the permeability ($R \propto \frac{l}{\mu}$)

(ii) the total flux is directly proportional to the permeability ($\emptyset \propto \mu$)

(iii) the flux density is directly proportional to the permeability ($B \propto \mu$)

EXERCISES

1. A copper ring with an average circumference of 10 cm. and cross-sectional area of 2 sq. cm. is wound uniformly with 1,000 turns of insulated wire. If the resistance of the coil is 500 ohms and it is connected across a 50V supply, calculate -

   (i) the reluctance,
   (ii) the total magnetic flux,
   (iii) the flux density, and
   (iv) the magnetising force in the magnetic circuit.

2. If an iron ring (μ = 500) is used instead of the copper ring in Exercise No. 1 above, calculate -

   (i) the reluctance,
   (ii) the total magnetic flux, and
   (iii) the flux density in the magnetic circuit.

3. An iron core 10cm. average length and 1.25 sq. cm. cross-sectional area, is wound with 500 turns carrying 600 mA. The flux set up is 25,000 lines of force. Calculate -

   (i) flux density;
   (ii) magnetising force; and
   (iii) relative permeability of the iron core.

9. AIR GAP IN MAGNETIC CIRCUIT

9.1 An air gap is introduced in the magnetic circuit of many items, for example, relays, induction coils, transformers, etc. This increases the total reluctance and reduces the flux density, for a given m.m.f.

9.2 Reluctances in series and parallel. When a series magnetic circuit consists of more than one type of material, or when the cross-sectional areas of various parts of the circuit differ, the total reluctance (RT) equals the sum of the individual reluctances (R1, R2, etc.).

$$R_T = R_1 + R_2 + \ldots$$

For magnetic paths in parallel.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

These formulas are similar to those for resistances in series and parallel.

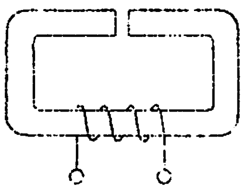
Note: When a small air gap is introduced in the iron core of an electromagnet, the cross-sectional area of the air can be assumed to be the same as that of the iron, that is, the flux passes straight across the gap.

9.3 Example No. 8. The electromagnet in Example No. 7 has an air gap of 0.1 cm. introduced in the magnetic circuit by a saw-cut through the iron. Calculate the new values of -

(i) total reluctance; (ii) total flux; and (iii) flux density in the magnetic circuit.

Solution:

Total reluctance (R_T) *= Reluctance of soft iron* (R_1 *+ Reluctance of air gap* (R_2)



$$\text{(i)}\quad R_T = \frac{l_1}{\mu_r l a_1} + \frac{l_2}{\mu_2 a_2}$$

$$= \frac{8 \times 10^5 \times 19.9 \times 10^4}{10^2 \times 10^3 \times 0.5} + \frac{8 \times 10^5 \times 0.1 \times 10^2}{10^2 \times 1 \times 0.5}$$

$$= 3.184 \times 10^6 + 16 \times 10^6$$

$$= 19.19 \times 10^6\ H^{-1} \text{ approx.}$$

$$\text{(ii)}\quad \emptyset = \frac{NI}{R} = \frac{6 \times 10^3 \times 40}{19.19 \times 10^3 \times 10^6} = 12.5\ \mu Wb$$

$$\text{(iii)}\quad B = \frac{\emptyset}{a} = \frac{12.5 \times 10^{-6}}{0.5 \times 10^{-4}} = .25T$$

Answers: (i) $19.19 \times 10^6 H^{-1}$; (ii) 12.5μWb; (iii) .25T

Compare these answers with those obtained in Example No. 7. Note that the air gap has increased the total reluctance, and decreased the total flux and flux density in the magnetic circuit.

9.4 Repeat the working out of Example No. 8 for an air gap of 0.25 cm. introduced in the magnetic circuit by a saw-cut through the iron. (The answers are $43.2 \times 10^6 H^{-1}$, 5.56 microweber and .1112 Tesla respectively.) Note the effect of the larger air gap on the total reluctance, total flux and flux density.

EXERCISES

1. An iron core 20 cm. long and 0.5 sq. cm. cross-sectional area is wound with 100 turns. Calculate the flux set up by a current of 2 amperes. (Relative permeability of the iron is 800).

2. An iron core 10.5 cm. long is wound with 300 turns of wire having a resistance of 25 ohms. Calculate the flux density in the core when the P.D. across the winding is 15 volts. (Assume the relative permeability of the iron to be 900).

3. (i) Using the formulas $F = \frac{\emptyset}{S}$ and $R = \frac{1}{\mu ra}$, derive a formula for μ_r in terms of $\emptyset$, F, l and a.

   (ii) A soft iron choke core 20 cm. long with a cross-sectional area of 0.5 sq. cm., is wound with 2,000 turns carrying 100mA. If the total flux in the magnetic circuit is 50 microweber, calculate the relative permeability of the soft iron.

4. (i) What is the reluctance of a soft iron core 20 cm. long, which has a cross-sectional area of 0.5 sq. cm. and a relative permeability of 1,000?

   (ii) What is the new value of reluctance when an air gap of 1 cm. is cut in the magnetic circuit?

5. (i) An iron core 100 cm. long with a cross-sectional area of 2.5 sq. cm. has an air gap of 5 mm. Relative permeability of the core is 1,500. Calculate the flux set up when the core is wound with 300 turns carrying a current of 250mA.

   (ii) What is the total flux when an air-gap of 0.5 cm. is cut in the iron?

6. A rectangular iron core 66 cm. long and with a cross-sectional area of 2.5 sq. cm. has an air gap of 5 mm. Relative permeability of the core is 1,500. Calculate the flux set up when the core is wound when 300 turns carrying a current of 250mA.

7. An iron core has a cross-sectional area of 3 sq. cm., the average magnetic path is 25 cm., and the air gap is 1 mm. If the relative permeability of the iron is 500, how many ampere-turns must be wound on the ring to set up a flux of 2.4 μWb in the magnetic circuit?

8. A magnetic circuit consists of a 100 cm. section of soft iron and a 1.5 cm. air gap. The cross-sectional area is 2 sq. cm. and the relative permeability of the iron is 1,000. Calculate -

   (i) the ampere-turns required to set up a flux density of 1 Tesla

   (ii) the number of turns carrying 2 amperes required to produce a flux of 200 microweber in the air gap.

9. An electromagnet has an air gap 6mm. wide and 20 sq. cm. cross-sectional area. The average length of the iron core is 50 cm., its cross-sectional area 10 sq. cm. and its relative permeability is 1,800. Calculate the current through 260 turns to set up a flux of 50 μWb in the air gap.

10. An iron core 60 cm. long and 12 sq. cm. cross-sectional area, is wound uniformly with 1,200 turns of insulated wire carrying a current of 1.5 amperes. Assuming a relative permeability of 700 for the iron, calculate the flux density in the ring before and after a gap of 1mm. is cut in it.

11. The following particulars are taken from the magnetic circuit of a relay -

Average length of iron circuit	20 cm.
Length of air gap between pole face and armature	1.5mm.
Cross-sectional area of iron circuit	0.5 sq. cm.
Effective cross-sectional area of air gap (pole face)	1.5 sq. cm.
Number of turns on core	8,000
Current through coil	50mA
Relative permeability of the iron	500

Neglecting leakage, what is the flux density in the air gap?

5. MAGNETISATION CURVES

Reference: ETP 0300 "Magnetism and Electromagnetism", Section 5.

1. B-H CURVES

1.1 When a low magnetising force (H) is applied to a magnetic material, the degree of magnetisation (flux density B) is also low. As the value of H is gradually increased from zero, the value of B also increases, slowly at first, then rapidly, then more slowly until magnetic saturation is reached. At this point, further increase in H produces no appreciable change in B.

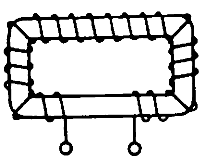
1.2 A graph showing the variation of flux density with magnetising force is called a B-H curve. This type of curve may be prepared by making a sample of the desired material into a closed loop, which is then closely wound with a known number of turns, and a known current is passed through the coil. This sets up magnetic flux in the sample, which can be measured with a fluxmeter.

If sufficient measurements and calculations (as shown in Example No. 1) are made, a B-H curve can be plotted.

1.3 EXAMPLE NO. 1. An iron core 20 cm. long and 0.5 sq. cm. cross-sectional area is wound with 6,000 turns of wire. When a current of 40mA passes through the coil, the total flux in the magnetic circuit is measured at 75 μWb. Calculate the values of H and B.

SOLUTION: $H = \frac{F}{l} = \frac{0.04 \times 6{,}000}{0.2} = 1{,}200\text{A/m}$

$B = \frac{\Phi}{a} = \frac{75 \times 10^{-6}}{0.5 \times 10^{-4}} = 1.5\text{T}$



ANSWERS: H = 1,200A/m; B = 1.5T.

1.4 As an exercise, calculate the new values of H and B when the current through the coil in Example No. 1, is reduced to 10mA and the total flux is measured at 57 μWb. (The answers are H = 300A/m and B = 1.14T).

2. PERMEABILITY CURVES

2.1 Just as the electrical conductivity of certain materials varies with the value of the e.m.f. applied to the material, so the magnetic conductivity (as indicated by the value of relative permeability) of a magnetic material is not constant but varies with different values of magnetising force and the degree of magnetisation of the material.

Using the values in paras. 1.3 and 1.4 -

When H = 1,200A/m, B = 1.5T

$$\mu_r = k\left(\frac{B}{H}\right) = \frac{8 \times 10^5 \times 1.5}{1{,}200} = 1{,}000$$

When H = 300A/m, B = 1.14T

$$\mu_r = k\left(\frac{B}{H}\right) = \frac{8 \times 10^5 \times 1.14}{300} = 3{,}040$$

2.2 For a particular set of operating conditions, if we know either the required value of flux density in the material or the magnetising force, the other can be read from the B-H curve, and the relative permeability calculated from the two. Permeability curves can then be plotted for the material, showing how μ_r varies for different values of H or B.

2.3 Permeability and B-H curves are sometimes plotted on the same graph.

EXCERCISE NO 1

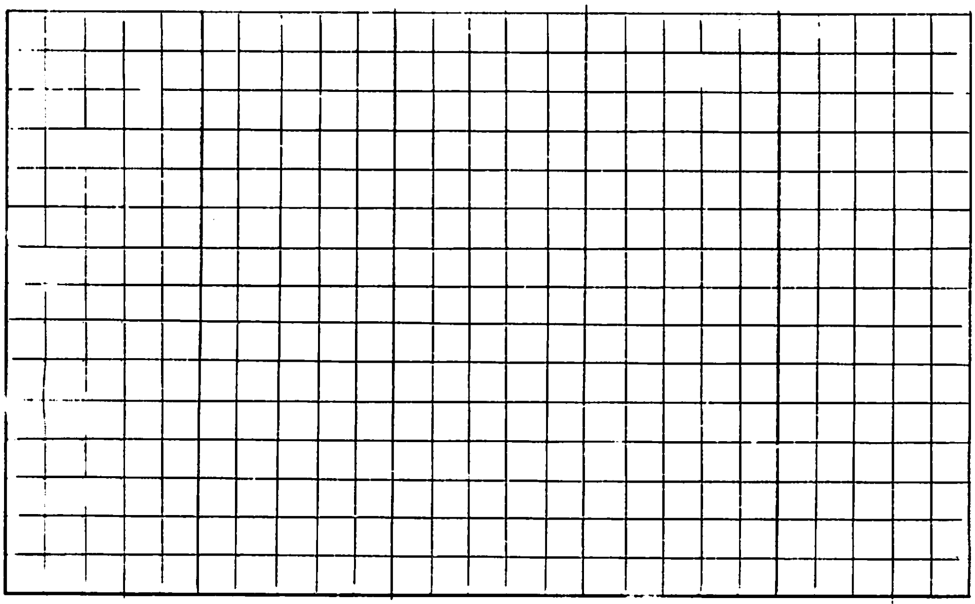
The following table shows measured values of H and B for a sample of soft iron. Using these values, plot a B-H curve showing how the flux density varies for different values of magnetising force.

H (A/m)	50	100	150	200	250	300	400	500	600	800	1000	1,200
B (teslas)	0.04	0.18	0.39	0.70	1.00	1.14	1.32	1.39	1.44	1.48	1.49	1.50

SOLUTION. Remember that graphs can be used to show relationships that cannot be seen clearly from a table of values.

(a) Select suitable scales and label the X and Y axes. In this example, a change in magnetising force (H) causes a change in flux density (B).

(b) Plot the relationship between B and H by drawing a smooth curve through the points obtained from the above table. (Note the three basic stages of magnetisation).



From the above B-H curve, find -

1. The flux density produced by a magnetising force of -
   (a) 80 amperes/metre; (b) 160 amperes/metre; (c) 450 amperes/metre.

2. The relative permeability when the magnetising force is -
   (a) 80 amperes/metre; (b) 160 amperes/metre; (c) 450 amperes/metre.

3. The magnetising force required to produce a flux density of -
   (a) 0.3 tesla; (b) 0.6 tesla; (c) 1.2 tesla.

4. The relative permeability when the flux density is -
   (a) 0.3 tesla; (b) 0.6 tesla; (c) 1.2 tesla.

EXERCISE NO. 2

Using the values for H and B given in Exercise No. 1, plot permeability curves, showing how the relative permeability varies for different values of magnetising force and flux density.

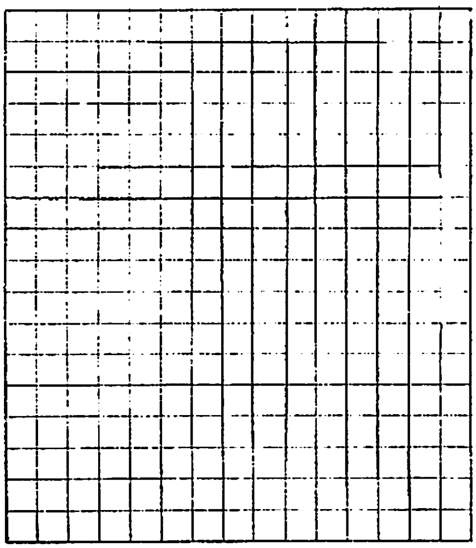
H (A/m)	50	100	150	200	250	300	400	500	600	800	1000	1,200
B (teslas)	0.04	0.18	0.39	0.70	1.00	1.14	1.32	1.39	1.44	1.48	1.49	1.50
$\mu_r = 8 \times 10^5 \times \frac{B}{H}$												

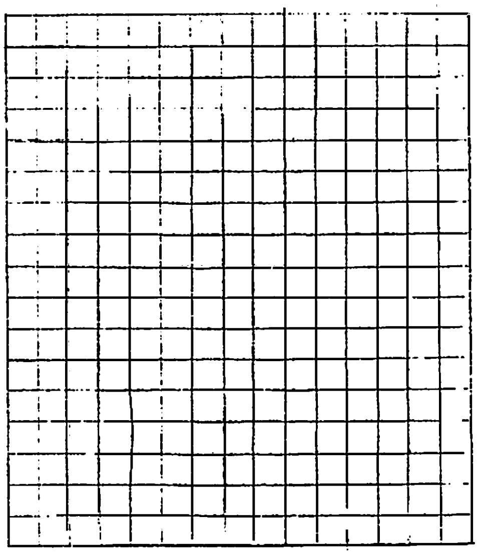
SOLUTION:

(a) Calculate and enter in the above table, the value of relative permeability for each set of values.

(b) On one graph, plot the relationship between μ_r and H by drawing a smooth curve through the points obtained from the above table.

(c) On the other graph, plot the μ_r-B curve by drawing a smooth curve through the points obtained from the above table.





From the above permeability curves, find -

1. The relative permeability when the magnetising force is -
   (a) 160 amperes/metre; (b) 550 amperes/metre.

2. The relative permeability when the flux density is -
   (a) 0.8 tesla; (b) 1.35 teslas.

3. The values of H and B when μ_r is 2,500.

4. The values of μ and B when H - 450A/m.

EXERCISE NO. 3

The following table shows typical values of H and B for a sample of permalloy (a nickel iron alloy).

H (A/m)	10	20	30	40	50	60	80	100	150	200
B (teslas)	0.02	0.1	0.25	0.42	0.57	0.69	0.76	0.78	0.80	0.81
$\mu = 8x10^5 x\frac{B}{H}$										

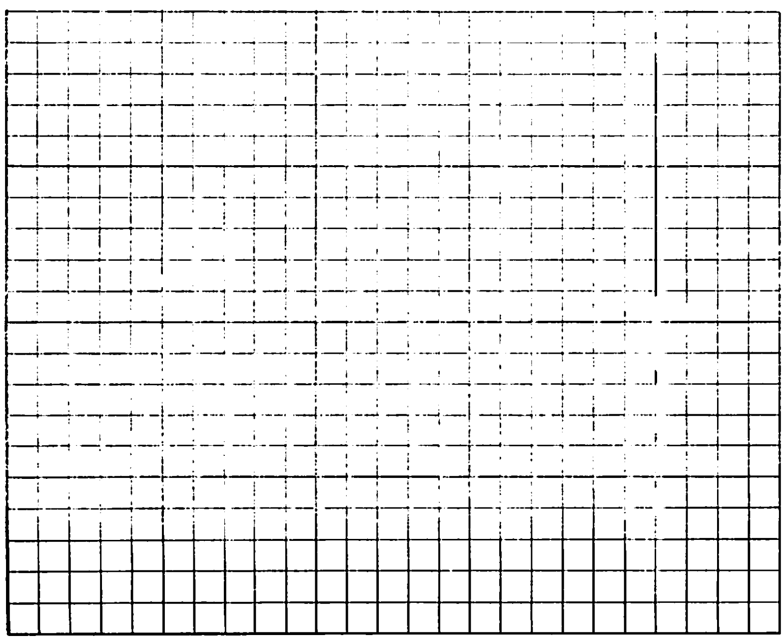
From the information in this table -

(a) Plot a curve showing how the flux density varies for different values of magnetising force. Label this, "B-H curve".

(b) Calculate and enter in the table, the value of μ_r for each set of values.

(c) Plot (on the same graph as the B-H curve) a curve showing how μ_r varies for different values of H. Label this, "μ_r-H" curve.

To prevent confusion between the two graphs, the values for B may be indicated on the left hand vertical axis, and the value for μ_r on the right hand vertical axis. Also, the B-H curve and B values may be shown in a different colour to the μ_r-H curve and μ_r values.

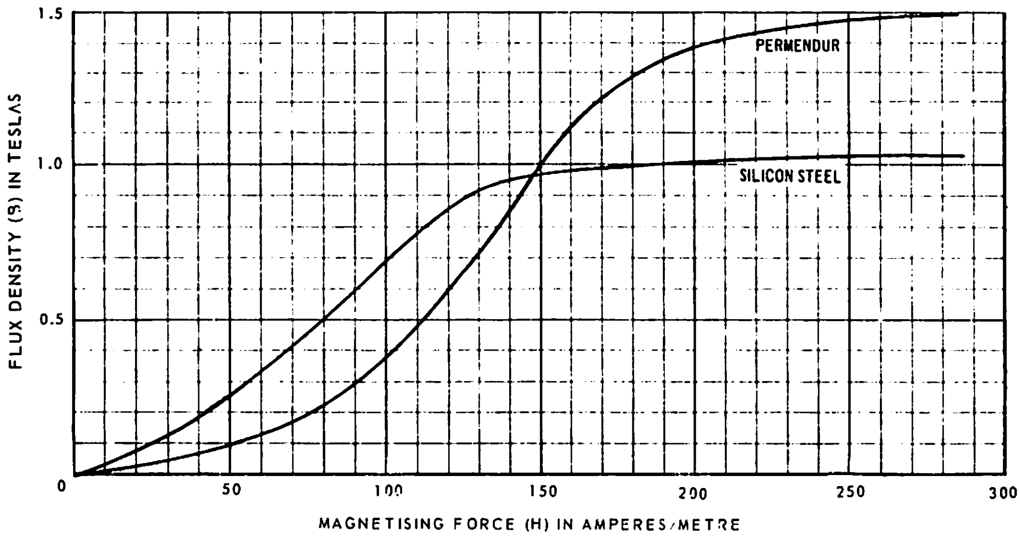


Note that μ_r reaches a maximum value as magnetic saturation is approached, and after that, falls off steadily.

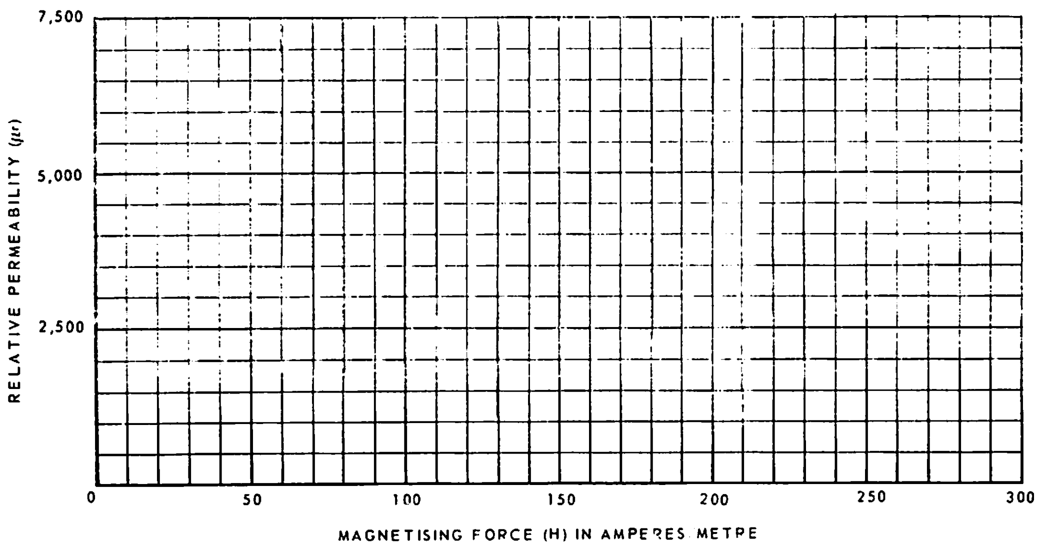
The values of H and B for which μ_r is a maximum may be found by drawing a straight line from the origin of the B-H curve to just touch the B-H curve.

In Exercise No. 3, what are the values of H and B for which μ_r is a maximum? What is the maximum value of μ_r?

EXERCISE NO. 4



Using the B-H curves above, plot on the following graph, curves showing how the relative permeability varies with magnetising force for silicon steel and permendur.



From information obtained on the μ_r curves, mark the section of the B-H curves where the relative permeability is greater than 5,000.

What are the values of H and B for which the μ_r of silicon steel is a maximum? What is the maximum value of μ_r?

What are the values of H and B for which the μ_r of permendur is a maximum? What is the maximum value of μ_r?

3. HYSTERESIS LOOPS

3.1 When magnetic material is subject to a complete magnetisation cycle, such as the core of a coil through which a pure sine wave current is passing, the flux produced lags behind the magnetising force. The effect is termed hysterisis and is represented graphically by a *hysteresis loop*.

During the magnetisation cycle some magnetism remains in the material when the magnetising force is zero. The remaining flux density (after saturation) is known as *remanence*.

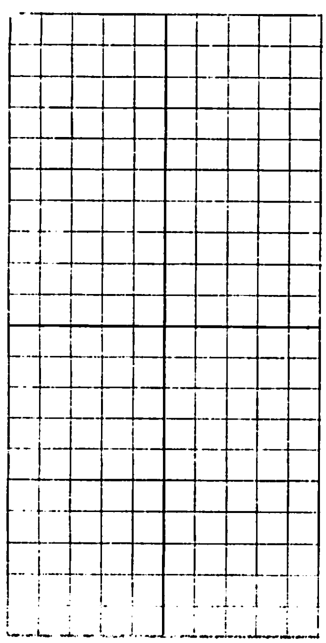
The magnetising force required to reduce the remanent flux density to zero is known as *coercive force*.

The area enclosed by the hysteresis loop of a substance gives an indication of the loss of energy (hysteresis loss) for each complete cycle of magnetism.

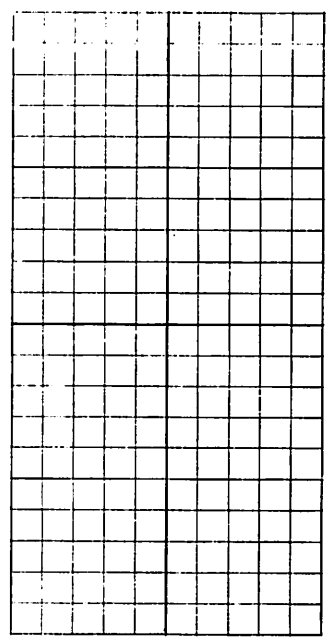
EXERCISE No. 5

Using the same scales draw graphs of the hysteresis loops of two magnetic materials from the following data, and indicate which has the greater hysteresis loss.

	Graph 1	Graph 2
B maximum (teslas)	1.4	1.4
H maximum (amperes/metre)	50	50
Remanence (teslas)	0.5	1.0
Coercive Force (amperes/metre)	5	20



GRAPH 1



GRAPH 2

6. TRANSFORMERS

Reference: ETP 0300, "Magnetism and Electromagnetism," Section 11.

1. TURNS RATIO

1.1 A transformer transmits electrical energy from one circuit to another by electromagnetic induction between two coils. The primary coil is connected to the source of supply. The secondary coil is connected to the load.

A step-up transformer has more turns on the secondary than on the primary. Generally, a step-down transformer has less turns on the secondary than on the primary. Some transformers may use two (or more) secondary windings to provide a step-up between the primary and one secondary winding, and at the same time, a step-down between the primary and the other secondary winding.

1.2 BASIC RULE AND FORMULA

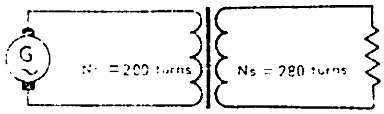
The turns ratio (symbol T) is the ratio of the number of turns (Ns) on the secondary winding to the number of turns (Np) on the primary winding).	$T = \frac{Ns}{Np}$

In some text books, turns ratio is expressed as the ratio of Np to Ns (T = Np/Ns). Therefore, to avoid misunderstanding when stating the turns ratio between the windings of a transformer, always indicate whether the transformer is a step-up or step-down type. For example:-

... for a transformer which has three times the number of turns on the secondary as on the primary, T = 3 (step-up)

... for a transformer which has one-third the number of turns on the secondary as on the primary, T = 3 (step-down).

1.3 EXAMPLE No. 1. A transformer has 200 turns on the primary and 280 turns on the secondary. What is the turns ratio?



SOLUTION: $T = \frac{Ns}{Np}$

$= \frac{280}{200} = \frac{1.4}{1}$

Answer: *Step-up turns ratio of 1.4.*

(If the number of turns on each winding were reversed, T would equal $\frac{1}{1.4}$ *signifying a step-down turns ratios of 1.4).*

EXERCISES

1. Calculate the turns ratio of a power transformer which has 13 turns on the secondary winding and 520 turns on the primary winding. (Indicate in your answer whether this is a step-up or a step-down transformer).

2. Calculate the number of turns on the primary winding of a step-up transformer which has a turns ratio of 3.2 and 400 turns on the secondary.

3. The primary winding AB of a transformer is tapped at point C. Winding AC has 450 turns and winding BC has 50 turns. The secondary has 250 turns. Calculate the turns ratio when the supply voltage is connected across (i) winding AB, (ii) winding AC.

4. A transformer with a primary winding of 400 turns has two secondary windings. Secondary winding No. 1 has 1,600 turns; and the turns ratio between the primary and the step-down secondary winding No. 2, is 4. Calculate:-

   (i) the turns ratio between the primary and secondary winding No. 1;
   (ii) the number of turns on secondary winding No. 2;
   (iii) the turns ratio of the transformer when the two secondary windings are connected in series.

2. VOLTAGE TRANSFORMATION

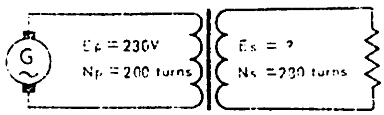
2.1 In a step-up transformer, the induced secondary voltage (Es) is greater than the applied primary voltage (Ep). In a step-down transformer, Es is less than Ep. The ratio Es/Ep is called the voltage transformation ratio.

2.2 BASIC RULE AND FORMULA.

In a transformer which is 100% efficient, the voltage transformation ratio (Es/Ep) is equal to the ratio of the secondary turns (Ns) to the primary turns (Np).	$\frac{E_s}{E_p} = \frac{N_s}{N_p}$

In a transformer which is less than 100% efficient, the voltage transformation ratio is correspondingly less than the turns ratio. In mathematical problems, unless otherwise indicated, the efficiency of a transformer is assumed to be 100%.

2.3 EXAMPLE No. 2. A transformer has 200 turns on the primary and 280 turns on the secondary. What voltage will it deliver from the secondary winding when the primary voltage is 230V?



SOLUTION: $\frac{E_s}{E_p} = \frac{N_s}{N_p}$

Therefore, $E_s = \frac{N_s \times E_p}{N_p}$

$= \frac{280 \times 230}{200} = 322V$

ANSWER: *322 Volts*

EXERCISES

1. Using the formulas given in paragraphs 1.2 and 2.2 derive formulas for:-

   (i) E_s in terms of E_p and T;
   (ii) E_p in terms of E_s and T.

2. A transformer is designed to supply an e.m.f. of 5V from the secondary winding when 230V is applied to the primary. Calculate:-

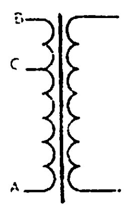
   (i) the voltage transformation ratios;
   (ii) the turns ratio.

3. A step-up transformer with a turns ratio of 3, has a voltage of 1.5V applied to the primary winding. Calculate the secondary voltage.

4. The secondary coil of a step-down transformer with a turns ratio of 20, has 100 turns; and the secondary voltage is 0.5V. Calculate:-

   (i) the voltage transformation ratio;
   (ii) the primary voltage;
   (iii) the primary turns.

5. The primary winding AB of a transformer has 480 turns and is designed to supply a secondary voltage of 240V when 240V is applied across winding AB. To enable 240V to be obtained from the secondary winding when 200V is applied to the primary, winding AB is tapped at point C. Calculate the number of turns on winding AC.



6. In Exercise No. 5, calculate the secondary voltage when:-
   (i) a supply voltage of 200V is connected across winding AB;
   (ii) a supply voltage of 24V is connected across wnding AC.

3. CURRENT TRANSFORMATION

3.1 In a transformer with a "closed" magnetic core, almost all the power delivered to the primary is transferred to the secondary. For all practical purposes, the power output from the secondary equals the power input to the primary, and the transformer is assumed to have an efficiency of 100%.

In a step-up transformer, the current (I_s) in the secondary circuit is less than the primary circuit current (I_p). In a step-down transformer, I_s is more than I_p. The ratio I_s/I_p is called the current transformation ratio.

3.2 BASIC RULES AND FORMULAS.

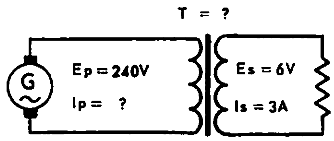
Power Output $(E_s\ I_s)$ = *Power Input* $(E_p\ I_p)$.
(Divide both sides by $E_s\ I_p$*, and cancel)*

$$\text{Therefore } \frac{I_s}{I_p} = \frac{E_p}{E_s} = \frac{N_p}{N_s} = \frac{1}{T}$$

Note that, in a transformer which is 100% efficient, the current transformation ratio is the inverse of the voltage transformation ratio and of the turns ratio.

3.3 EXAMPLE No. 3. The primary of a power transformer is connected to a 240V power supply. The secondary delivers 6V at 3A. Assuming 100% efficiency, find -

(i) the current in the primary;
(ii) the turns ratio.



SOLUTION. *(i)*

$$E_p\ I_p = E_s\ I_s$$

$$\textit{Therefore} \quad I_p = \frac{E_s\ I_s}{E_s}$$

$$= \frac{6 \times 3 \times 1{,}000}{240} = 75\ mA.$$

(ii) Turns ratio

$$\frac{N_s}{N_p} = \frac{I_p}{I_s}$$

$$= \frac{75}{3{,}000} = \frac{1}{40}$$

This is a step-down transformer with a turns ratio of 40.
(Check this value by using the formula for turns ratio in terms of the voltage ratio).

Answers:- (i) 75mA; (ii) step-down turns ratio of 40.

EXERCISES

1. A transformer draws 20 watts from a line. What is the secondary current if the secondary voltage is 8 volts?

2. A power transformer delivers 1.5A at 6.3V. If the primary voltage is 230V, find:

   (i) the primary current, (ii) the power input.

3. A transformer with 2,400 turns on the primary and 480 turns on the secondary, draws 8.5A from a 230V line. Calculate:-

   (i) the secondary current;
   (ii) the secondary voltage;
   (iii) the power output.

4. A step-down transformer with a turns ratio of 4, draws 1.8A. Find the secondary current.

5. Find the primary voltage in a transformer when the secondary voltage is 230 volts, the primary current is 3.65 amperes and the secondary current is 0.1 ampere.

4. IMPEDANCE TRANSFORMATION.

4.1 For maximum power transfer from one circuit to another, the impedances of the two circuits must be either equal or "matched". If the two circuits have unequal impedances, a transformer may be used as an impedance matching device between the two circuits. The turns ratio of the matching transformer depends on the values of the impedances to be matched.

A step-up impedance matching transformer is used when the load impedance (Z_s) of the secondary is greater than the impedance (Z_p) offered by the primary to the supply under load conditions. A step-down transformer is used when Z_s is less than Z_p. The ratio Z_s/Z_p is called the impedance transformation ratio.

4.2 BASIC RULES AND FORMULAS. In a 100% efficient transformer-

$$Z_s = \frac{E_s}{I_s} \quad \text{and} \quad Z_p = \frac{E_p}{I_p}$$

$$\text{Dividing } Z_s \text{ by } Z_p,\ \frac{Z_s}{Z_p} = \frac{E_s}{I_s} \div \frac{E_p}{I_p} = \frac{E_s}{E_p} \times \frac{I_p}{I_s}$$

$$\left(\text{but } \frac{E_s}{E_p} = T \quad \text{and} \quad \frac{I_p}{I_s} = T\right)$$

$$\text{Therefore, } \frac{Z_s}{Z_p} = T^2$$

The impedance transformation ratio is equal to the square of the turns ratio.	$\frac{Z_s}{Z_p} = T^2$
The turns ratio is equal to the square root of the impedance transformation ratio.	$T = \sqrt{\frac{Z_s}{Z_p}}$

Derive a formula for Z_s = ... (in terms of Z_p and T).
Derive a formula for Z_p = ... (in terms of Z_s and T).

EXERCISES

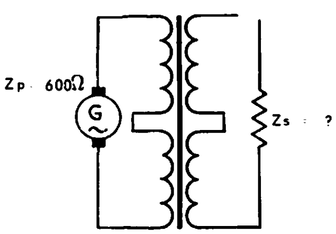
1. What is the impedance transformation ratio of a transformer which has 4,000 turns on the primary and 6,000 secondary turns?

2. An impedance matching transformer with a step-down turns ratio of 2, has a 600 ohm generator connected to the primary winding. What value of load impedance must be connected across the secondary to ensure maximum power transfer from the generator to the load?

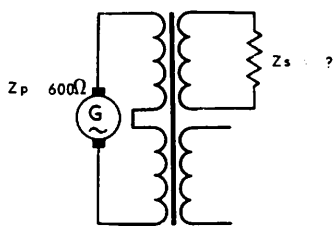
3. Calculate the turns ratio of the transformer used to match:-

   (i) a telephone line impedance of 600 ohms to a 50 ohm load impedance.
   (ii) a line impedance of 600 ohms to an item of equipment with an impedance of 240 kilohms.
   (iii) a 5,000 ohm primary impedance to a 600 ohm secondary. If the primary has 20,000 turns, how many are required on the secondary?

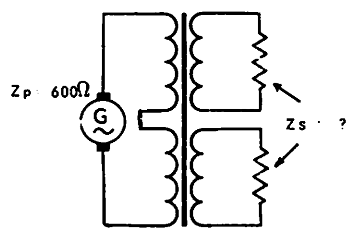
4. In each of the following diagrams, calculate the load impedance (Z_s) for maximum power transfer. (Note:- Each of the four windings has the same number of turns.) Hint: Remember that total output power dissipated in secondary load(s) equals power input to the primary winding.



(i)



(ii)



(iii)

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15	.18
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17	.23
18	.26
19	.28
20	.30
21	.32
22	.34
23	.36
24	.38
25	.40
26	.42
27	.43
28	.45
29	.46
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46	.66
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54	.73

NO.	LOG.
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99	.99

ANTI-LOGARITHMS

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ANTILOG.	NO.
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